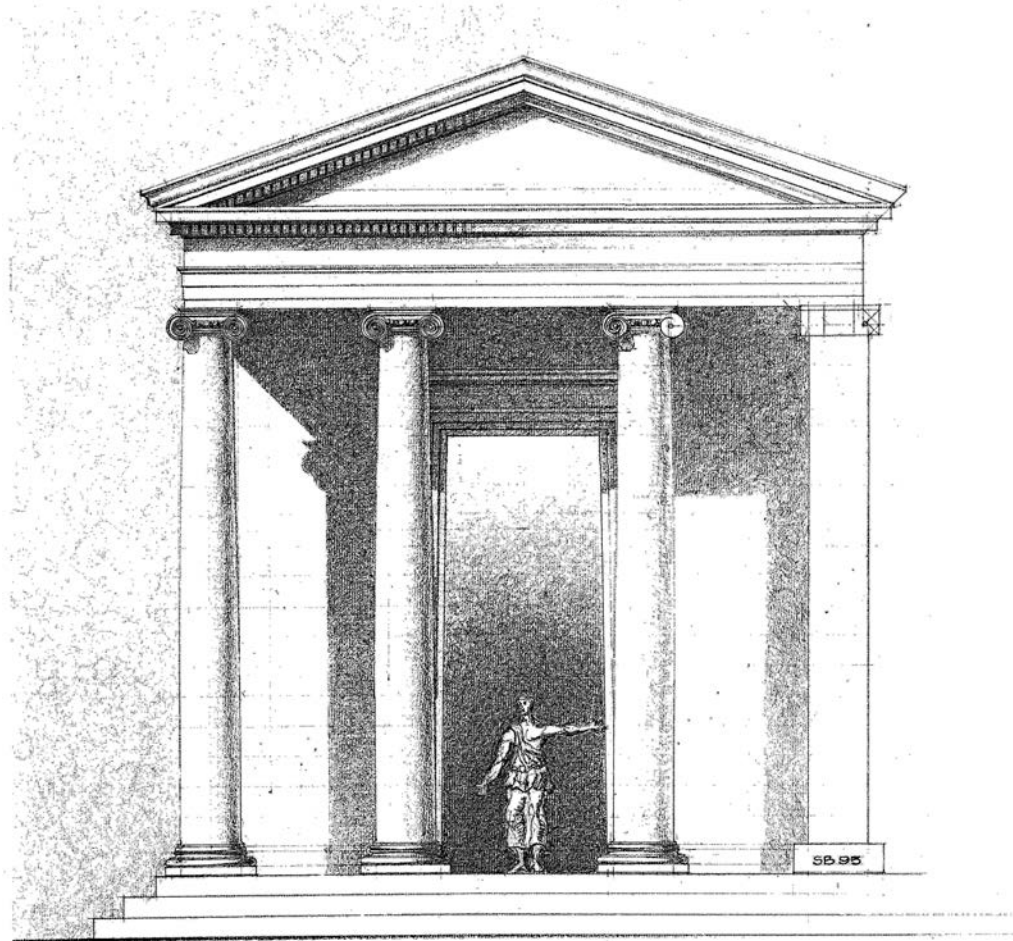


# A Theory of Proportion in Architecture and Design

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presented at the  
**Institute of Classical Architecture & Art**  
20 West 44th Street New York NY 10036  
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### COURSE DESCRIPTION

The class provides an overview of the concept of proportion as a design tool in traditional architecture; presented in the form of lectures and demonstrations. The content includes:

**Session 1** - an explanation of the idea of symbolic or qualitative number; an introduction to Pythagorean and Platonic numerical philosophy; the nature of beauty, its relation to number, the derivation of the ancient musical octave and its use as a guide to harmony.

**Session 2** - a discussion of the Golden Section, its mathematics, geometry, relation to philosophy, particularly its role as a geometrical “logos” and the connection of all these ideas to the numerical-geometrical canons of classical architecture.

**Session 3** - These ideas are applied to historical archetypes, such as the four, six and eight column porticos, through demonstrations using arithmetic, geometric, and harmonic ratio methods of application to achieve design outcomes that approach beauty. The course may be taken as an introduction to these ideas or as a review for those who have some background with the subject.

### LEARNING OBJECTIVES

the course will enable students to:

1. appreciate the nature of beauty and the role it plays in individual spiritual development.
2. understand number in a symbolic rather than mathematical sense and how this connects aesthetics to aspects of ancient philosophy.
3. understand the nature of rational ratios, composed of whole numbers, and transcendental ratios such as the Golden Section which are not composed of whole numbers, as an approach to achieving the beautiful.
4. use geometric, arithmetic and harmonic methods of application of proportion for analysis of historical designs and to create new designs.

### RECOMMENDED READING

Allen, Jon, **Drawing Geometry**, Floris, 2007, Edinburgh

*A good introduction to constructing polygons, providing a foundation for further study.*

Bass, Steve, **Beauty Memory Unity**, Lindisfarne Books, 2019

*A more full explanation of the subjects of this course.*

Bragdon, Claude, **The Beautiful Necessity**, Theosophical publishing, 1978

*A modern view of architecture as an expression of an occult Unity. A good introduction.*

Critchlow, Keith, **Time Stands Still**, Gordon Fraser, 1979

*Pythagorean analysis of stone circles and ancient symbolic consciousness.*

Fletcher, Rachel, **Infinite Measure**, George F Thompson Publishing, 2013

*A clear, elegant contemporary presentation of the principles of sacred geometry, particularly the Golden Section, as they relate to traditional and modern architectural design.*

Lawlor, Robert, **Sacred Geometry**, Thames & Hudson, 1982

*A key statement of geometric principles for our generation, written with dramatic flare. Follow the workbook style exercises into the more profound discussions.*

Olsen, Scott, **The Golden Section: Nature's Greatest Secret**, Wooden Books, 2006

*Popular, contemporary exposition of the GS - good background for the designer.*

Schneider, Michael, **A Beginner's Guide to Constructing the Universe**, Harper, 1994

*An exposition of the perennial tradition written for the contemporary general reader. A good book to start with. Also see [www.constructingtheuniverse.com](http://www.constructingtheuniverse.com)*

Wittkower, Rudolf, **Architectural Principles in the Age of Humanism**, Norton, 71

*An often cited reference and an excellent discussion of Pythagorean musical theory behind the architecture of Alberti and Palladio.*

## Opening

### Personal Intro

Good day. Thanks to all of you attending and to those at the ICAA who invited me. I'm Steve Bass, an architect in practice in New York City. I studied modern architecture at Pratt Institute from 1965 to 1970; my teachers had studied with Walter Gropius at Harvard in the post-war era. When I began to practice on my own I found that my performance varied in quality and I did not have a 'bottom line' to evaluate design. After looking through modern writings I began to study the classical in the mid nineteen seventies. Reading Vitruvius motivated me to study the elements with architect Alvin Holm. As Vitruvius refers to Plato and Pythagoras I sought a teacher for these matters and found Dr. Keith Critchlow at the Royal College of Art in London. This course is a synthesis of the teaching of those two persons.

### Handout description

Everything we will be saying and demonstrating today is in this handout. The handout begins with a theoretical statement, then illustrates a series of applied methods. It should be treated as a workbook. There is also a book that grew out of this course called 'Beauty Memory Unity'

### subject intro

Proportion is the use of number and geometry as design tools. But of all the subjects relating to classical architecture, proportion seems to be the most opaque and difficult aspect for the contemporary mind to grasp. Many good designers deny they use it

Classical architecture can be said to be a 'rational' art; that is, it can be described in words and follows a logical structure. It may thus be said to be Apollonian, fully enlightened and rooted in text. Apollo is the divinity in charge of the muses who govern the arts.

Many aspects of classical architecture are Apollonian, and we are not going to argue here that it does not have this character. However, to understand proportion we will need to travel another path, one more likely associated with Hermes, the trickster, or Dionysus, who uses intoxication, than with the rational Apollo. It's a path no less significant or important, just less brightly lit.

Entering the subject through its rational mode we note that all rational studies or sciences begin with axioms. These are foundational principles required for the rational development of the science to follow. Interestingly, they are statements or definitions which are assumed. They are not to be proved within the science based on them, and they may not be provable at all, like the existence of the point in geometry.

In ancient times any rational logical form of study was called a science. Thus in consideration of classical architecture as a rational, scientific form of study I propose three axioms -

Axiom 1 - Classical architectural compositions are built up from a kit of pre-existing parts, known as the elements. These parts have a grammar or syntax that allows them to additively create more complex forms, such as the 'orders' or genres, or as the archetypal four column portico.

Axiom 2 - The elements and archetypes are described in a series of reference books, known as the 'canon'. These are used as sources and references for design. The canons are not identical and therefore should only represent the beginning of one's work or study, they are not to be taken literally or prescriptively. Anyone who has designed 'from the book' has probably done it only once.

Application of these two axioms allow classical architecture to achieve a higher purpose, beyond mere appearances, to be used rhetorically, to tell a story, known as 'historia' in the fine arts.

These axioms also allow the design to achieve a 'character'; a state in which the building 'speaks' in an intelligible language about what it is, who made it, and who it is meant to serve.

In the canonical references the elements and archetypes are described by number and geometry, that is, by proportion.

Thus in a classical composition Proportion gives every element some intelligible relation to every other part and to the whole allowing historia or character to emerge.

Axiom 3 - Most importantly, the goal of composing in a classical manner is the achievement of the possibility of the experience of beauty.

Alberti's defined beauty as 'a state where nothing can be added or removed from the design except to its detriment'. In such a state the design achieves what he calls 'concinatus', where the elements are harmonically, that is proportionately, related. This is an 'exoteric' definition, dealing with objects. It does not define Beauty itself, but lets designers know when their work approaches it.

If beauty is the goal, and number is the tool, there must be an intelligible relation of number to beauty. Exploring this relationship, focused on classical architecture will be the primary theme of this course. The goal being to allow designers to begin to utilize this relationship.

### Rational Ratios

To see how this relationship could work and more importantly, to allow us to impart a vital quality to our current work we will need to turn to an earlier, pre-modern vision of number - and here we must leave the Apollonian path.

The philosophy associated with Pythagoras provides us with such a vision. Pythagoras was a philosophical teacher in the seventh and sixth centuries BC. He studied with Thales who sent him to Egypt for further study. After 22 years in Egypt and 12 years in Mesopotamia he returned to Greece and began to teach. These numbers are symbolic, 22 with Thoth-Hermes and 12 with astronomical astrology. Egypt and Mesopotamia are the precursor civilizations of the West. Pythagoreanism was more than an intellectual teaching. It was a way of life associated with vegetarianism and Orphic musical religion.

For our purposes here, Pythagoreans considered that there were three kinds of number -

quantitative number - the number of counted things - 2 apples + 2 apples = 4 apples - but if you have 2 apples and 2 oranges you have a category problem. This brings forth -

mathematical number - abstract magnitudes and operations - we can say  $2 + 2 = 4$  and don't have to say 4 of what. This must be strictly rational. We are familiar with these two kinds of number. The Pythagoreans had a third kind of number -

qualitative number - in which each number is a self-contained image having a unique identity - it is the idea of oneness, the quality of being one, Unity; as opposed to one object or a magnitude of one.

To illustrate this concept of qualitative or symbolic number, which is difficult for us today to envision, we may use as an example a version of an ancient creation myth, a story about how things came to be.

### Figure 1. A Geometrical Creation Story

Here is a story of creation, an ontogeny, to illustrate the concept of qualitative or symbolic number, **figure 1a**.

**1. The Monad.** Creation begins with the point of origin, embodying the quality of oneness, called the Monad. It represents the state of transcendent unity which is the necessary starting point of our world of multiplicity. This point contains a fundamental mystery - it has location without dimension. Though the point contains everything in potential it cannot be perceived with the senses as it is unextended in dimension. It is represented in arithmetic by the number one and in geometry by the dot. The first shape is the circle. It is an inflation of the point that allows its potential to be actualized. This first stage of our story of coming into being may be thought of as that of transcendent Unity. When taken as an applied unit of measure consistent with its character of sameness such a key unit of measure is called the module.

**2. The Dyad.** If unity contains all, everything manifest and unmanifest, then it must also contain its own opposite, division or separation from unity. For such a separation to occur there must be a direction prior to any movement. This initial form of extension is known variously as the ray of creation, lightning flash, world tree or axis mundi. Geometrically this is the line. This movement is not an external separation but is division of unity from within. It is represented here by two equal circles centered on the axial ray. Twoness, called the Dyad, is the ontological level of moral choice; the good versus the not good. For Pythagoreans, the dyad symbolized polar opposites such as: limit-unlimited; odd -even; one-many; rest-motion; same-different. Order is connected to the idea of limit, for recognizable order must fall within some limit. Disorder and chaos are unlimited.

**3. The Triad.** Any two separate things must have a relationship. This requires threeness or the Triad. For sensible perception or any act of communication to take place, there must be at least a sender, a receiver and a medium of transmission. For any kind of physical structural stability, a spatial triangulation is required. As it contains the beginning, middle and end the Triad is the ontological level of the symbolic, of the image and of the individual soul. We represent the triad in our diagram by placing a third circle between those of the same and the different. This circle governs the creation of the equilateral triangle, the first regular polygon.

**4. The Tetrad.** A fully manifest object must have at least three dimensions, length, width and height. These three intervals are marked off by a minimum of four points: \*length \* width \* height\*. Fourness or the tetrad is the ontological level of manifestation. It may be visualized as a crossing of dyadic opposites, such as warp and weft which cross to create a fabric, or male and female polarities which must be crossed to allow physical birth. Geometrically, the Tetrad may be represented by the square. It is in this sense that we still use the expression 'the four corners of the world.'

Our ontogeny takes us from the mysterious point of emergence to full manifestation in four stages. The building blocks of this creation myth are the qualitative aspects of number and geometry. In the fifth century BC, about a century and a half after the life of Pythagoras, this qualitative aspect of number was taken up by Plato, most notably in the dialog of 'Timaeus'. In that work Plato weaves a wonderfully complex and subtle creation myth using the four ancient numerical sciences: Arithmetic - number in concept, Geometry - number in space, Music - number in time, and Astronomy - number in space and time.

These four sciences correspond to the ontological stages of our creation myth. In **figure 1b** each of the subjects is viewed in four stages, suggesting that they are each cosmologies of their own. The physical world of the Platonic elements also known as the sub-lunary world, corresponds to the

cosmological stages as well. The elements - fire, air, water and earth - are listed in order of energy and density; fire being the most energetic and least dense, earth being the opposite.

In the 'Timaeus' and the 'Republic' Plato employs the sciences of number to model what he calls the 'world soul'. A geometric image of this concept is shown in **figure 1c**. Though Plato uses the sphere to represent the world soul, for brevity's sake, we represent it with the circle, in effect, a sphere in elevation. Soul or psyche may also be thought of as 'mind' or 'consciousness'. The diagram indicates that the individual soul is similar and concentric to the world soul; it is 'analogos'. The function of psyche is to project Nous, Idea or Form, into matter, which has no inherent form.

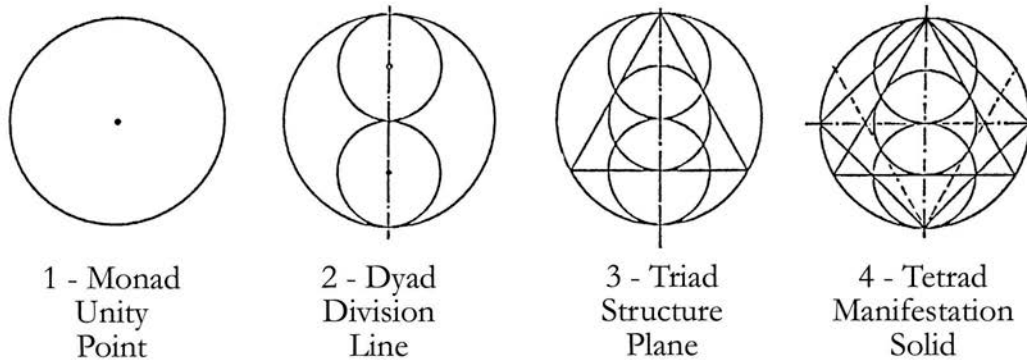
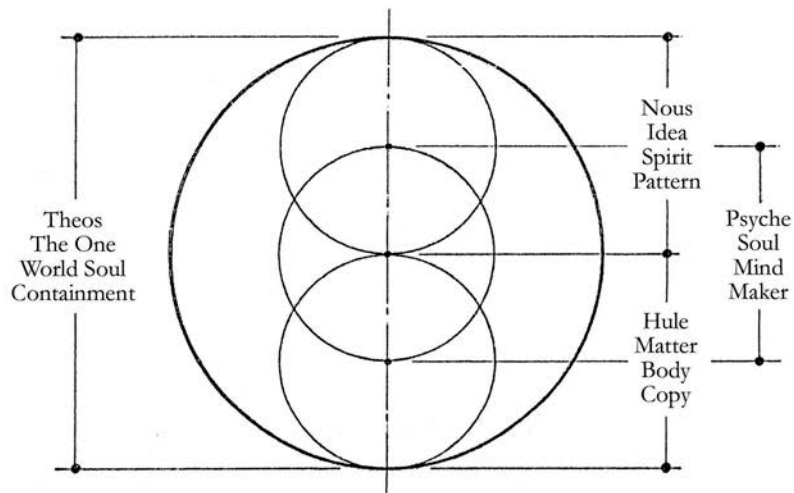


Figure 1a - The four stages of the Geometrical Creation Story

Symbol	Ontological Stage	Subject	Number	Geometry	Music	Astronomy	Platonic Element
	Unity	Number	1 Monad	Point	Fundamental 1 : 1	Light	Fire
	Division	Geometry	2 Dyad	Line	Octave 2 : 1	Dark	Air
	Structure	Music	3 Triad	Plane	Fifth Note 3 : 2	Gnomon	Water
	Matter	Astronomy	4 Tetrad	Solid	Fourth Note 4 : 3	Orientation	Earth

1b - Correspondences of the Four Mathematical Subjects of Plato



1c - Geometric Diagram of the Platonic Psyche

### Figure 2. The Musical Octave

One of the great discoveries attributed to Pythagoras is that relationships between consonant sounds may be expressed in terms of number. Ancient music used a natural or untempered octave and composition was based on four notes known as the tetrachord. When expressed as ratios of string length the tetrachord is; 1:1 - the fundamental, 2:1 - the octave, 3:2 - the fifth note, and 4:3 - the fourth note. Thus a string twice as long as the string that produces the fundamental produces a consonant or harmonic sound one octave lower.

These relationships are illustrated in Ptolemy's 'Harmonics', **figure 2a**, with the diagram called the 'Helikon', named after the mountain where the Muses lived. One must imagine an actual instrument with a square frame, four strings and a diagonal bridge. The length of the string to create the fourth note of the octave is given by the point of intersection of the square's diagonal and its semi-diagonal. The length of the string to create the fifth note is given by the intersection of the semi-diagonal and the centerline of the square. This geometrical pattern which divides the sides of the square into two and three parts can also be used to create architectural elements.

In **figure 2b** the vertical diameter of the outer circle is seen as a string. The inner circle above gives the fundamental interval. The inner circle below is subdivided by the same pattern as the 'Helikon'. If we think of the outer circle as the world soul the diagram may be seen as a kind of cosmic monochord

A four column temple portico may be constructed on the same geometrical net as the musical octave, **figure 2c**. Both utilize the division of the square into thirds. The height and position of the inner column is given by the one-third point, which also gives the position of the 4th note. The depth of the entablature is one-fourth of the column height and is equivalent to the interval of the whole tone, between the 4th and 5th note. The lower diameter of the column is one-eighth its height, and one-twelfth of the width of the portico, thus relating the diameter to both the whole tone and the semi-tone. The height of the pediment is equal to the entablature and completes the square. This construction conforms to the canon of Vignola and comes within 5% of the canon of Vitruvius.

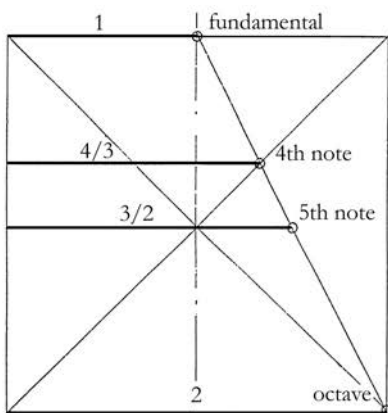


Figure 2a - Ptolemy's Helikon

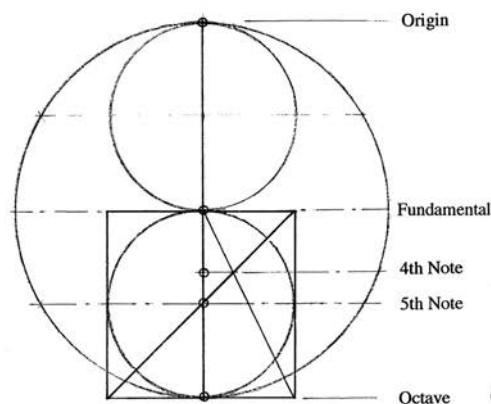


Figure 2b - Geometrical Construction

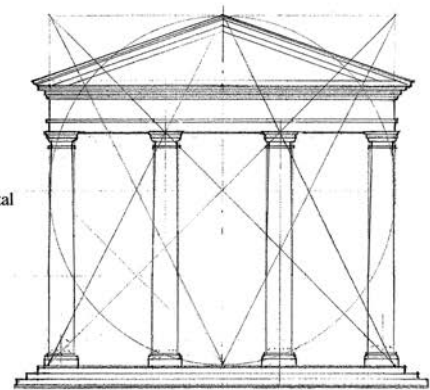


Figure 2c - A Tetrastyle Portico

**Figure 3. A Tetrastyle Portico**  
using harmonic division of the square

To look closer at the construction of the portico of **figure 2c** the design process may be divided into four stages which recapitulate the stages of our creation myth, **figure 3**.

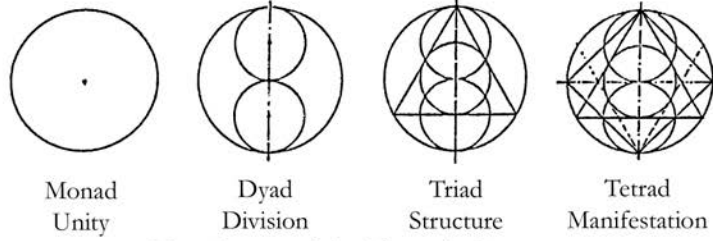
**1. Unity** - Begin by restating the Pythagorean geometrical model-monad, dyad, triad, tetrad. From the inner tetrad construct an outer square, a material representation of unity, which will be the limit of the height and width of our design.

**2. Division** - The essential scission of unity. The square is divided by means of its diagonals and semi-diagonals. The intersection of these two sets of diagonals divides the sides of the square at the 1/3rd points. These points will mark the centers of the two inner columns and also their height.

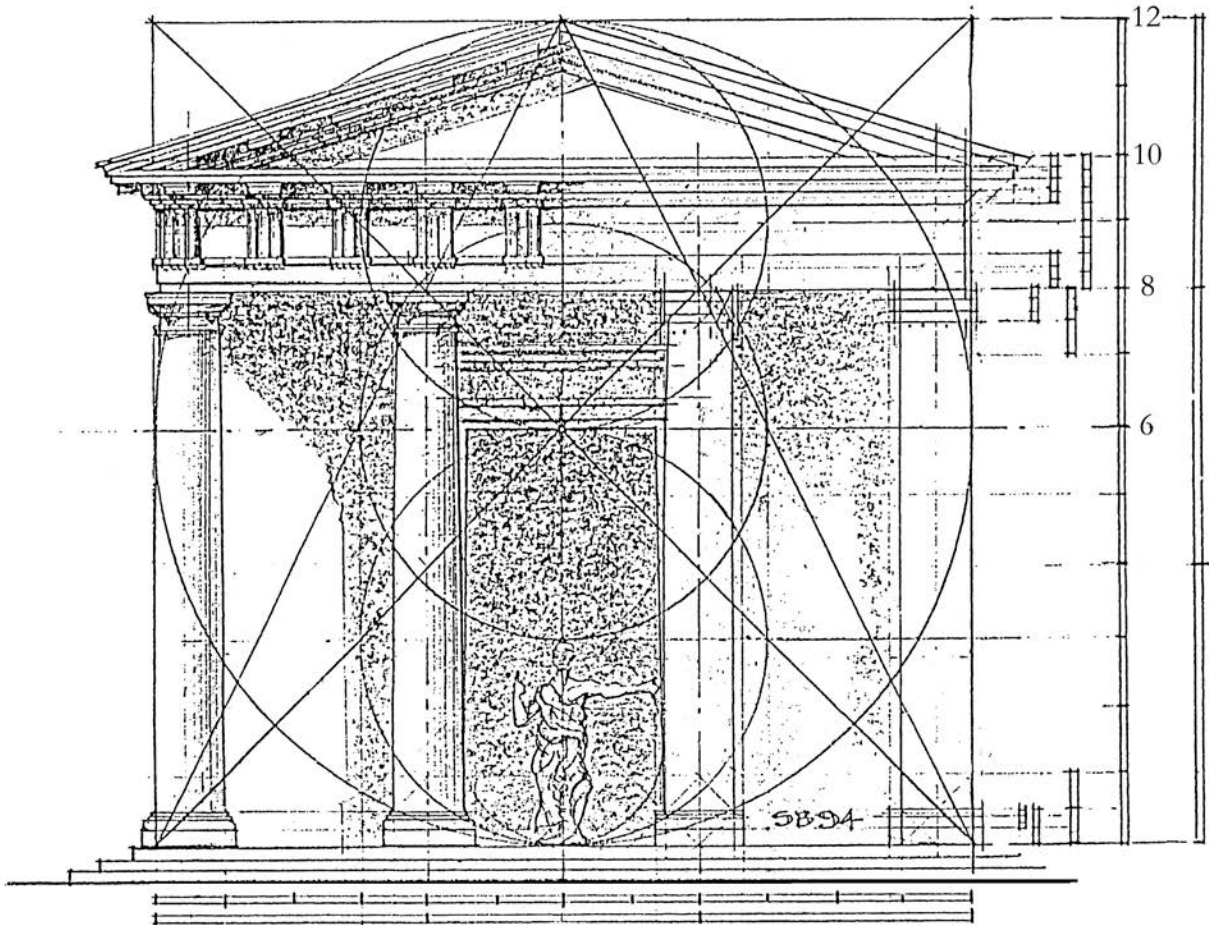
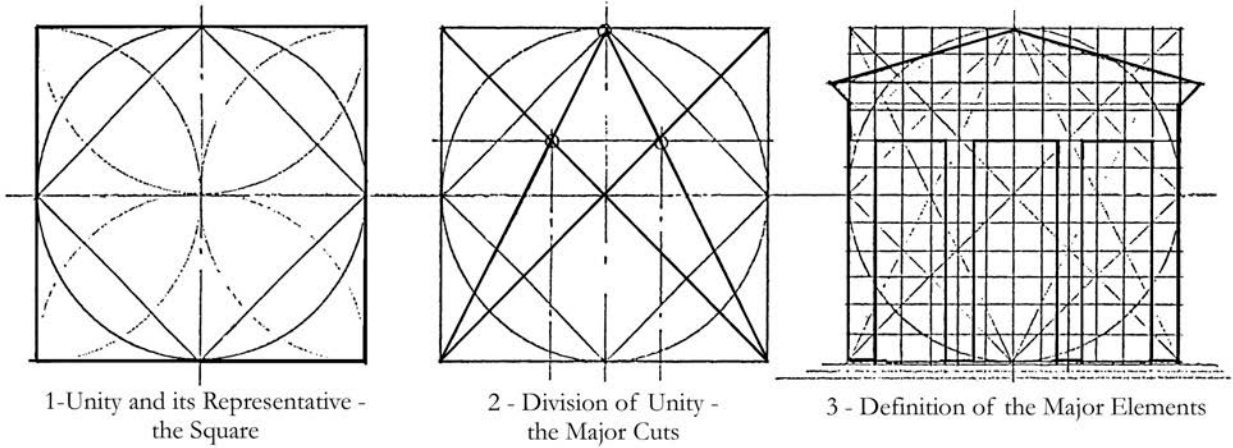
**3. Structure** - The disposition and demarcation of the major parts of the design. By the canon of Vignola, if the height of the columns, including base and cap, is divided into four units, the height of the entablature is one additional unit. The lower diameter of the columns will be 1/8th of their height. In our design this is 1/12 of the side of the enclosing square. The sizes of other elements are derived from the lower diameter. Here we may see an integration of incommensurable geometry and the linear measures of the canon. The decimal equivalent of 1/3 is .333...; that of half the side of the square is .500. The difference between them is 1/6 or .166... half of which is 0.083..., or 1/12. Thus geometrical manipulation alone could have been used to size and position the major elements.

**4. Manifestation** - Full articulation of the details and ornaments. The order selected for this design is Doric. Of the three Greek orders, Doric may be most associated with the Apollonian or rational function; the initial phase of the triadic cycle, the proceeding or coming forth. According to Vitruvius it may be selected to represent senior male divinities such as Zeus, heroes like Herakles, or certain female divinities such as Athena. The portico also references Apollo as overseer of the Muses in that it is based on the same geometry used to construct the intervals of the ancient musical tetrachord, shown in **figure 2**. In **figure 12** the Doric capital is constructed on the same geometrical pattern as that of the facade as a whole.





Four Stages of the Platonic Ontogeny



4 - Detailing and Ornamenting the Elements

Figure 3

### Figure 4. An Ontogeny of the Root Powers

The ontogenic story, or creation myth told with arithmetic and geometrical images, in **figure 1**, is not unique nor is it an isolated instance. Indeed, there may be an indefinite number of such stories. The earlier story was used to model the emergence of the 'rational ratios', a series comprised of small whole numbers that are related to the musical octave. Presented here is a variant of that story which gives rise to a series of 'transcendental' or 'trans-rational' ratios, that is, ratios that are not composed of whole numbers but of components that are not determinable in rational units. The most commonplace of such ratios is that of the circle's diameter to its circumference, known as pi,  $\pi = 1:3.14\dots$ . The three dots after the figure indicates that the sequence of numbers does not have a knowable termination but continues indefinitely. Pi has been calculated, not just to millions of digits but to millions of pages of digits. All the computers in all the world, working for all the time there is will not pin it down.

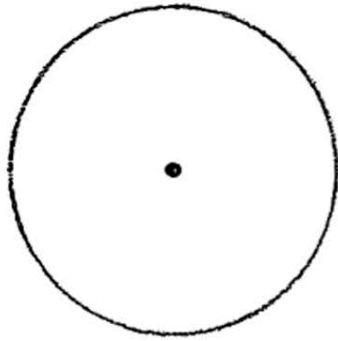
These trans-rationals have a special character in that we can see directly the relation of the factors when they are drawn graphically, but we cannot fully express these relationships in rational mathematical units. We may take this stubborn 'fact of life' as an indicator that the rational aspect of the human being, while important in the expression and communication of knowledge, is not the whole story. To put it another way, the existence of the trans-rationals suggests that there may never be a fully rational explanation of existence and the cosmos, or as the humorist puts it, to 'life, the universe and everything'. This may account for the persistence of the power of myth. Certainly, in the stimulation of remembrance the artist must, in some way, enter the realm of the trans-rational.

As earlier, the story begins on the level of the Monad, with the mysterious point of emergence and its inflation into the circle, **figure 4a**. We project the initial ray of creation or lightning flash, **figure 4b**. In the earlier story, **figure 1**, this stage of Division, from the Monad to the Dyad was represented by placing two circles within the circle of Theos, each half the diameter of the larger circle. Both these circles, symbolizing the Same and Different, Idea and Matter, are aspects of 'Theos', and they divide within Theos, thus preserving a single unified cosmos. In the earlier story they continued their division until their circumferences were tangent at the center of the circle of Theos, forming a figure eight or meniscus.

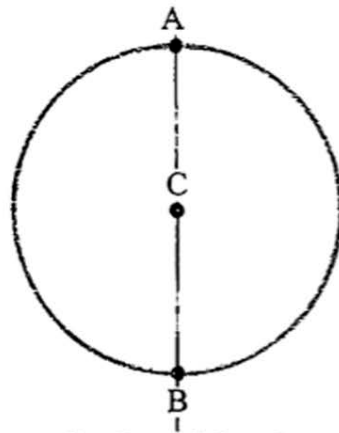
In this story we must envision, **figure 4c**, that in response to the ray of creation the circle of the Same begins as the circle of Theos itself and withdraws upwards, decreasing in diameter, while maintaining its tangency at point A; and simultaneously the circle of the Different, also beginning as the circle of Theos, withdraws downwards diminishing in diameter while maintaining its tangency at point B. In our earlier story the differentiation continued until the circles of the Same and Different were tangent at the center, point C. Here we stop the differentiation at a place where the center of one circle lies on the circumference of the other. The centers of these two circles may be found, **figure 4d**, by walking the radius AH around the circumference. From C and D find point G. Draw lines GE and GF to find H and J, the centers of the circles of the Same and the Different.

The football shaped area of overlap of the circles of the Same and Different, **figure 4e**, represents the area of Psyche, Soul or Mind; the Triadic level of our ontogenic story. This shape is called the Vesica Pisces or Vessel of the Fish. If we connect MN and PQ the lines will meet at C, the center of the circle of Theos, and will cross at a right angle,  $90^\circ$ , **figure 4f**. If we take this as symbolizing the Tetradic or fourth ontological stage we will have recapitulated the steps in **figure 1**. Because intervals MN, MP and NP are all radii, shape MNP is an equilateral triangle. If the distance MN, from center to center is taken as 1, the distance from point to point, PQ is equal to the  $\sqrt{3} = 1.732\dots$

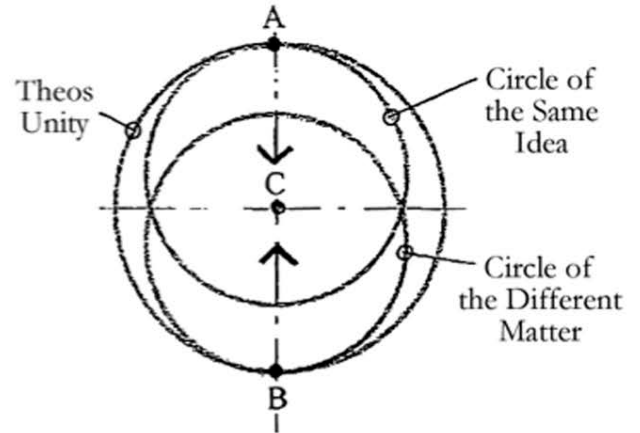
Figure 4 - The Root Rectangle Family  
An ontogeny of the Vesica Piscis



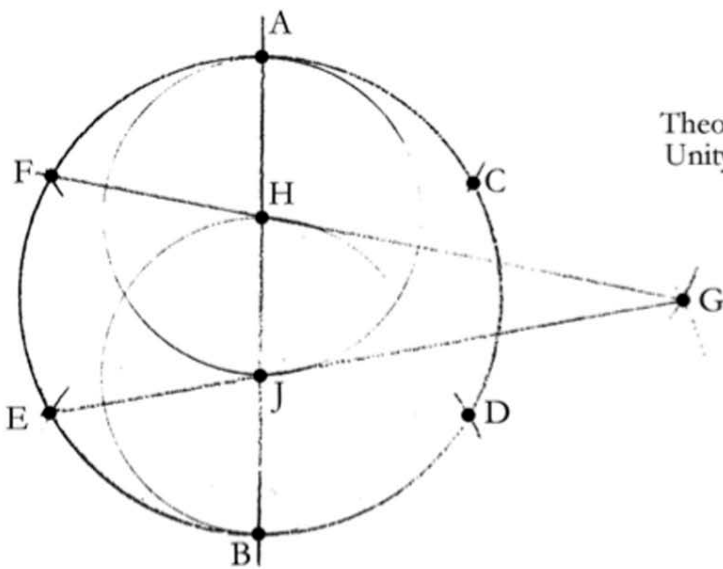
4a - Circle of Theos



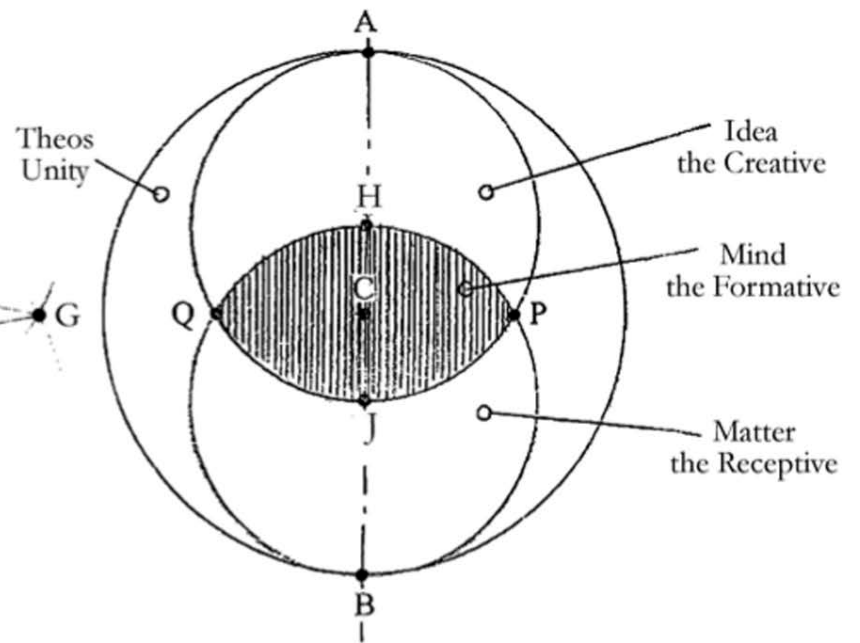
4b - Ray of Creation



4c - Same & Different separate Within Theos



4d - Finding the Centers of the Vesica Piscis



4e - The Vesica Piscis

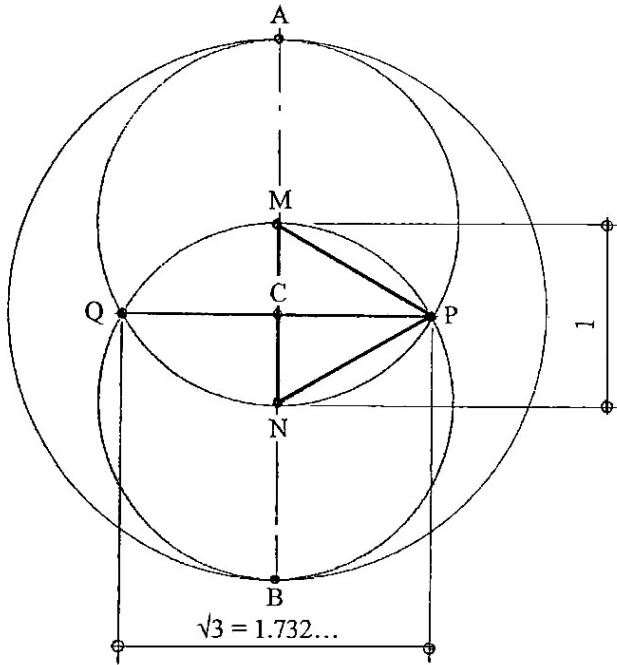
The Vesica Pisces played a role early Christianity. As a graphic sign it is said to have marked Christian meeting places during the time when the religion was illegal. In a deeper way it is related to Christianity symbolically as the birth of Jesus occurred at the start of the Piscean age, when the sun on the Spring equinox began to rise in the constellation Pisces. For the previous two thousand years it had been moving through the constellation Aries; it is now about to enter the constellation Aquarius. The Christian era has thus taken place in the Piscean, or water age. Water imagery surrounds Jesus; he is the 'fisher of souls', his disciples are fishermen, he walks on water, and feeds the multitude with loaves and fishes. In John 21:11 the resurrected Christ bids fishermen to cast their net into the water and causes the miraculous catch of 153 fish. This may be a reference to the Pythagorean 'measure of the fish' or 265/153, a whole number approximation of the  $\sqrt{3}$ .

The Vesica Pisces may also symbolically represent the yoni or female birth organ. As the Vesica Pisces gives birth to the initial form, the equilateral triangle, it may also give birth to any regular polygon. The 'trick' being how far to separate the two radii, MP and NP. Stated another way, the task is to find the center of the circle which will circumscribe the desired polygon. Construct a semicircle, **figure 4g**, centered at C with radius CM and find point R, the center of the square's circumscribing circle. Extend a line through MR until it intersects the circle at S. Repeat the process from N through R to point T. MNST is a square, and the diagonal  $MS=NT=\sqrt{2}=1.414\dots$

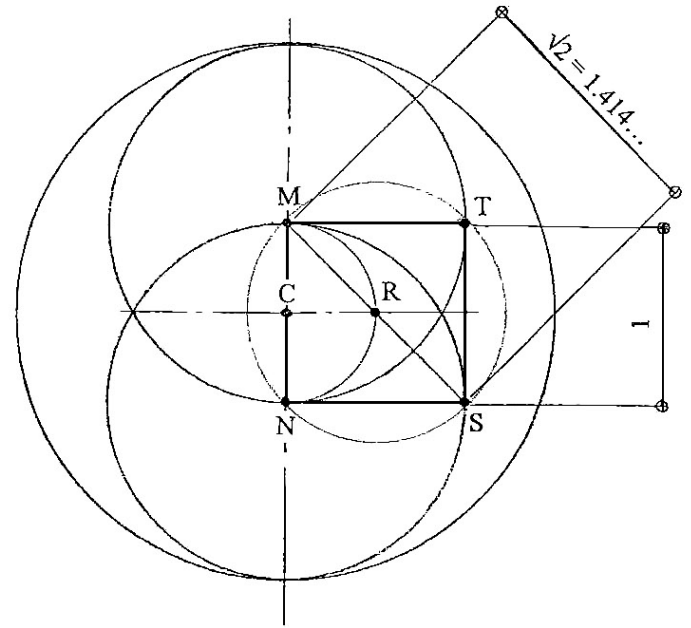
The triangle and square have been constructed on the right side of the Vesica Pisces. Constructing a mirror image square on the left side, **figure 4f**, gives rectangle UTSV. If  $ST = 1$ ,  $VS = 2$ ; and 2 is the  $\sqrt{4}$ . By the Pythagorean theorem,  $a^2 + b^2 = c^2$ , the diagonal of double square UTSV is equal to the  $\sqrt{5} = 2.236\dots$ . This represents a movement beyond the story in **figure 1**. If the first four stages represent our descent from Unity into Materialization, the Pentad, fiveness, symbolizes the balance point, or the 'turning point' of our return.

In the Pythagorean myth the individual soul begins its journey by projecting into the cosmos from the divine realm as a ray of light. As the psyche approaches the earth it passes through the shells of the seven planets, acquiring character. When it passes below the level of the moon it begins to take on material form. At last it reaches the earth where physical birth occurs. But according to Plato<sup>1</sup> the effect of the journey into incarnation is to cause the psyche to, in effect, become disoriented, forget its identification with divine unity and to identify with the body. Thus our earthly work is one of remembrance or in Platonic terms 'anamnesis'; a necessary precursor for psyche's return journey to the divine. This return from multiplicity to unity is symbolized in the ratio called phi,  $\Phi$ , or the Golden Section.

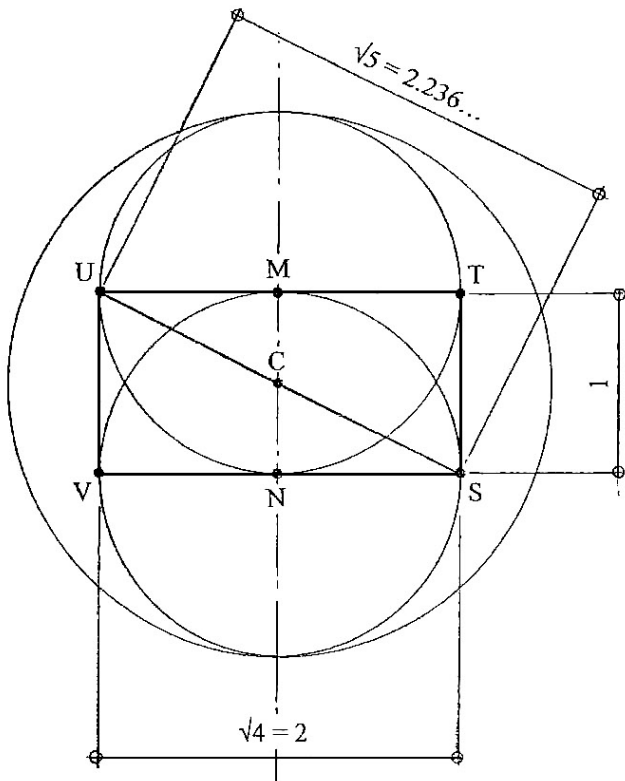
In **figure 4j** the interval UC is 'applied' along vertical axis AMC to find point W.  $UC=\sqrt{5}/2=1.118\dots$ , and  $MC = 1/2 = .5$ , thus the numerical value of  $\phi = .618\dots$ . If we draw a line through W parallel to UM, and extend UV the two lines will meet at point Y creating a phi rectangle. Point Y falls on the circle of Theos, the Divine. This is because in triangle YWC,  $WC=1.118\dots$  and  $YW = 1$ ;  $YC = 1.118\dots^2 + 1 = 2.249$ ,  $\sqrt{2.249\dots} = 1.499\dots$  and  $AC = 1.5$ . Thus phi has geometrically reenacted the metaphor of Platonic remembrance and returned us to Unity via a journey through multiplicity.



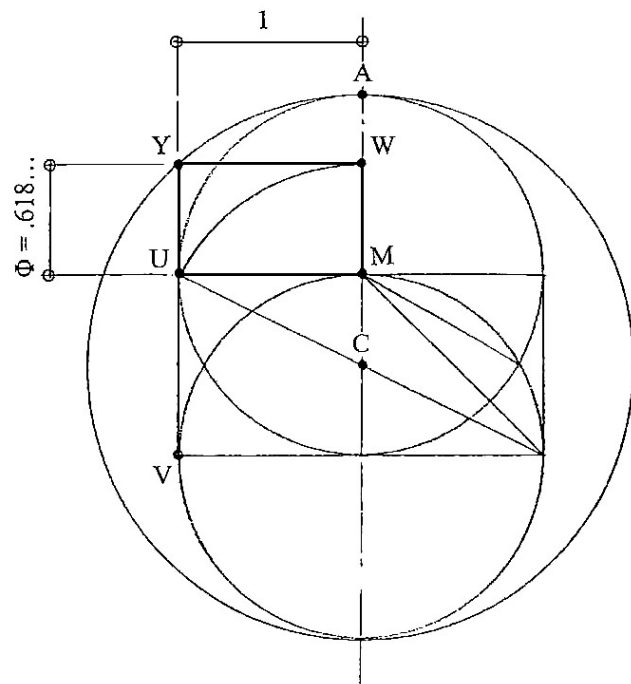
4f - The Equilateral Triangle and the  $\sqrt{3}$



4g - The Square and the  $\sqrt{2}$



4h - The Double Square and the  $\sqrt{5}$



4j - Phi Returns to Unity Through Multiplicity

Figure 4 - The Root Rectangle Family  
The root rectangles, phi  $\Phi$ , and the return to unity

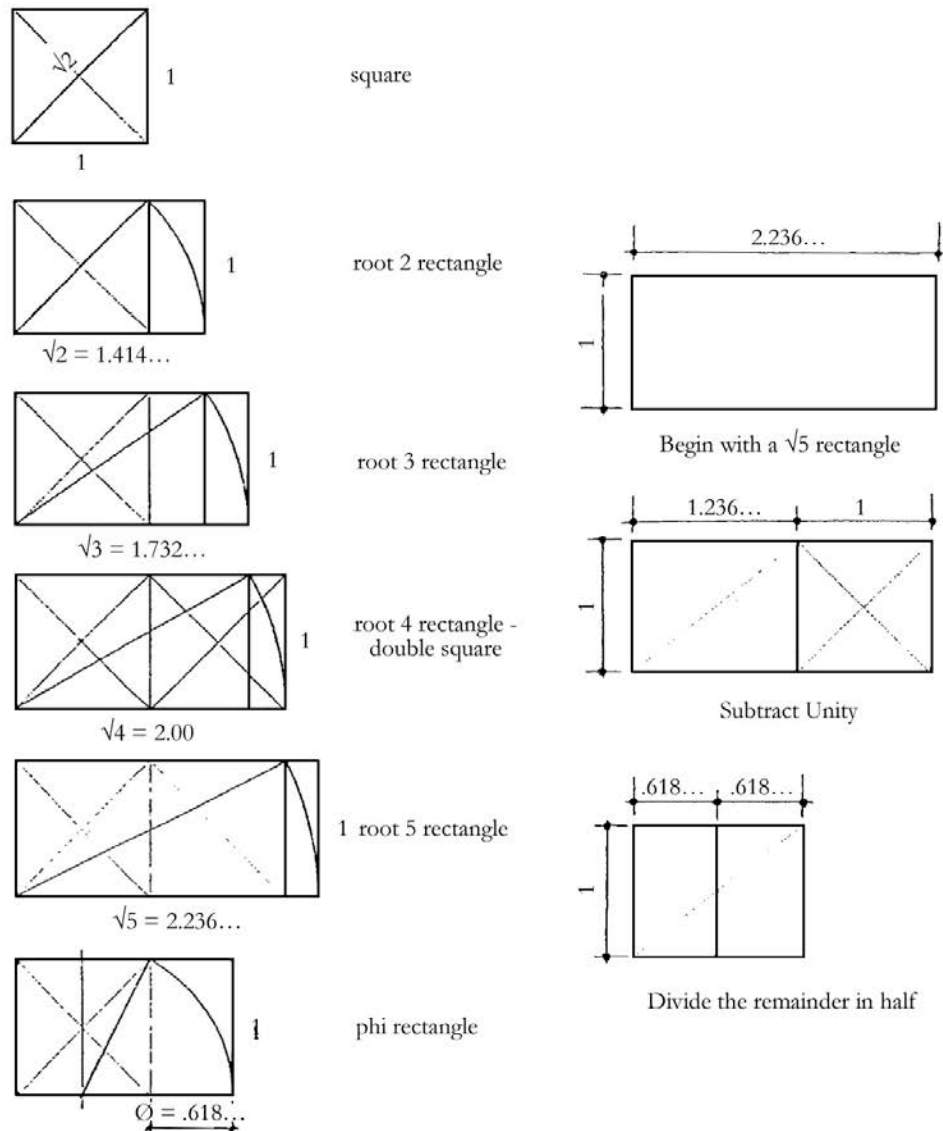
### Figure 5. Properties of the Root Rectangles

The root powers form a series of rectangles exhibiting a family-like resemblance, starting with the square, **figure 5a**. The diagonal of a square whose side =1 is  $\sqrt{2} = 1.414\dots$ . Rotating the diagonal down to the horizontal, or as geometers might say, 'applying' the diagonal upon the extended edge, produces a rectangle whose sides are in the ratio of 1:1.414..., called a ' $\sqrt{2}$ ' rectangle.

In a remarkable aspect of the nature of number the diagonal of a  $\sqrt{2}$  rectangle =  $\sqrt{3}$ , 1.732... Continuing with the process it is even more remarkable that the diagonal of a  $\sqrt{3}$  rectangle is equal to  $\sqrt{4} = 2$ , producing the double square. This constitutes a kind of 'crossover' point between the rational and the trans-rational ratios. Continuing another step, the diagonal of a double square =  $\sqrt{5} = 2.236\dots$

Although we could continue indefinitely, design tradition has not found it necessary go beyond the  $\sqrt{5}$  rectangle, most likely because this is a source of the Golden Section, as  $\phi = [\sqrt{5}-1]/2$ . A geometrical version of this algebraic procedure, **figure 5b**, starts with a  $\sqrt{5}$  rectangle. Subtract a square, then divide the remaining interval by 2, leaving a  $\phi$  rectangle.

In a superimposed view of the root rectangle family, **figure 5c**, we see the 'crossover points'

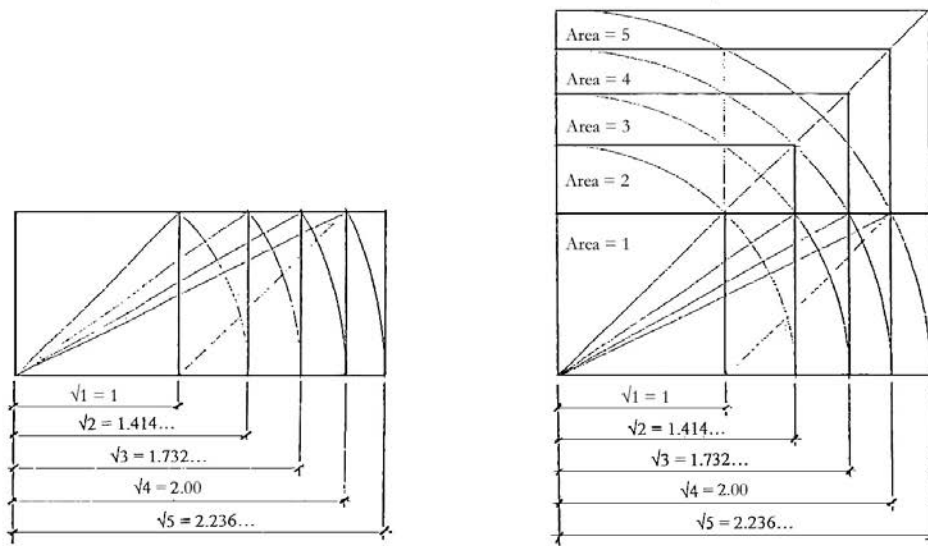


5a - The Root Rectangles - Individual Construction

5b - Geometrical Algebra of Phi

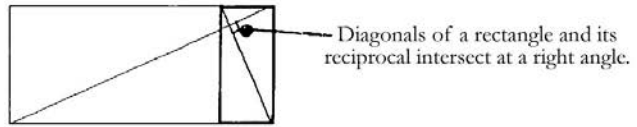
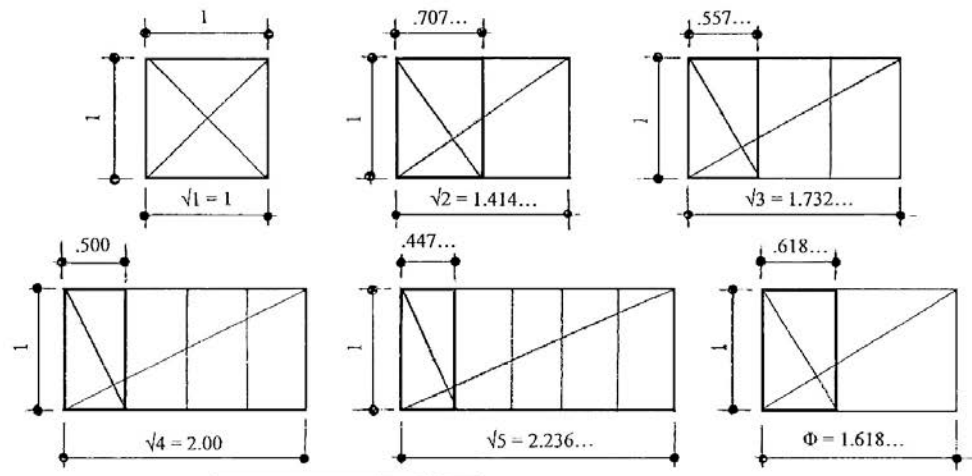
between the rational and trans-rational ratios marked by the square and the double square. An extension of this diagram, **figure 5d**, illustrates the area relationships of the family. The square, initial representative of unity, has an area of 1; a square whose side is equal to the  $\sqrt{2}$  has an area of 2; and so on.

All rectangles have a 'reciprocal'; **figure 5e**, that is, if the short side of a rectangle is 1 and the long side is 2, if we then set the long side equal to 1 the short side is now equal to  $1/2$ ; or if the sides are 1 and  $\sqrt{2}$  the reciprocal is 1 by  $1/\sqrt{2}$ , or  $1:.707...$ . The reciprocal of the  $\sqrt{3}$  rectangle is  $1:1/\sqrt{3} = 1:.557...$ ; of the  $\sqrt{5} = 1:1/\sqrt{5} = .447...$ . The reciprocal of a  $\sqrt{2}$  rectangle divides its parent rectangle into two equal parts; the reciprocal of a  $\sqrt{3}$  rectangle is  $1/3$ rd of its parent; and so on. The reciprocal of a  $\Phi$  rectangle is another  $\Phi$  rectangle plus a square because  $1 : .618... = 1.618... : 1$ . The square is its own reciprocal. The diagonal of a rectangle intersects the diagonal of its reciprocal at a right angle, called by Le Corbusier the 'place of the right angle'.



5c - The Root Rectangles Superimposed Construction

5d - Area Relations of the Root Rectangles



Diagonals of a rectangle and its reciprocal intersect at a right angle.

5e - Reciprocals of the Root Rectangles

### Figure 6. The Golden Section

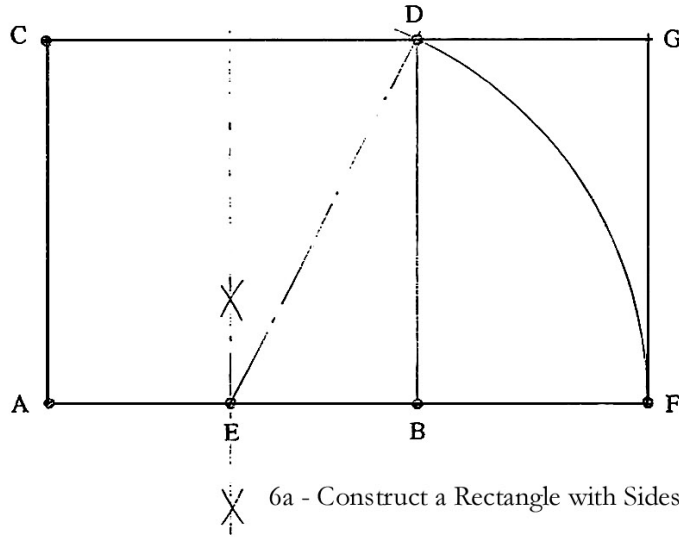
In classical times mathematical and geometrical descriptions of the universe were contained within the philosophy of the Pythagoreans. They considered the sensible cosmos to be 'ana-logos' that is, 'like the logos'. Although the term 'logos' has many meanings, for our purposes here it is taken to be 'a numerical pattern used by the Divinity to divide Unity in such a way as to allow for its reunification'. This reunification or remembrance of Unity, which Plato called 'anamnesis' is associated with the experience of Beauty. The captivating quality of Beauty, Plotinus tells us in *Ennead 1.6*, is the outer aspect of the attractive power of Love. The rush of Erotic emotion and captivation we feel in the experience of Beauty, according to this theory, results from the Soul's identification of Unity within apparently separate objects. This identification is a function of Soul.

In the *Republic* Plato directs designers, whether of nations or buildings to *"turn their eyes upwards and downwards ... first look at absolute justice, beauty and temperance, and again at the human copy."* He adds that the designer should *"mingle and temper the various elements of life into the image of a man"*, that is, an image like ourselves, a copy of the Divine pattern. This pattern is the Logos expressed as number.

The most prominent numerical pattern associated with the idea of Logos was known in ancient times as the 'extreme and mean ratio', called the Divine Proportion in the Renaissance and the 'Golden Section' [ratio, proportion or division] in more recent times. Today it is denominated by the Greek letter phi,  $\Phi$ . In nature  $\Phi$  governs the proportions of many living forms. In Pythagorean philosophy it is associated with the magical animation of matter. It is also the key to the Aristotelian theory of 'mimesis', which may be broadly stated as 'art imitates life' or more specifically visual art imitates organic life. However, mimesis should be in the service of anamnesis.

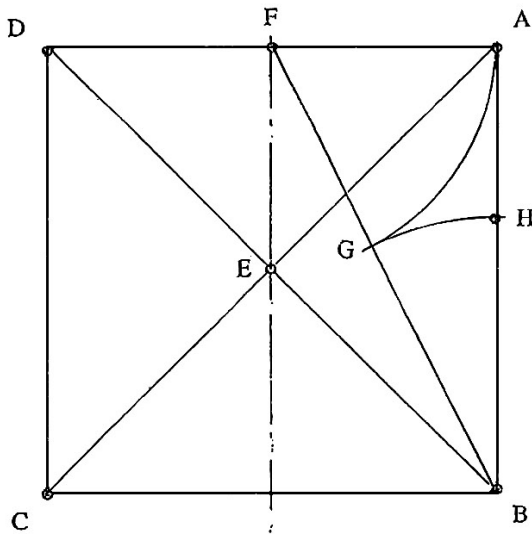
Symbolically  $\Phi$  represents return to Unity through division. The portico of **figure 3**, in its twelve-fold division, may be said to have used the symbolic numerical powers of 3 and 4, as  $3 \times 4 = 12$ . But  $3 + 4 + 5$  also = 12. Among many discoveries ascribed to Pythagoras is that the numbers 3, 4, and 5, taken as intervals of length, are the smallest whole numbers that will form a right triangle whose sides satisfy the famous equation  $a^2 + b^2 = c^2$ . This '3-4-5 archetype' forms the basis of an ontogeny of its own, that of the triadic cycle. Four may be taken here to represent the two-fold power, as  $2^2 = 4$ . It is associated with the proceeding aspect of the cycle in that the  $\sqrt{2}$  that governs the doubling of the area of the square, symbolizing the essential division of unity, the first step toward manifestation. The three-fold power is associated with the maintaining aspect through the relation of the  $\sqrt{3}$  to the altitude of the equilateral triangle, symbolic of both mediation and structure. The five-fold power represents the returning aspect through its relation to  $\Phi$ , which is mathematically derived from the  $\sqrt{5}$ .





1. Given line AB
2. Construct square ABCD
3. Bisect AB to find E.
4. Draw arc with center at E and radius ED to find point F on the extension AB.
5. Construct a perpendicular on F and extend CD to find point G.  $AC : AF = 1 : 1.618\dots$

6a - Construct a Rectangle with Sides in the Ratio of the Golden Section



1. Given line AB. Construct square ABCD.
2. Draw diagonals AC and BD to find center E. Draw centerline EF
3. Draw semi-diagonal FB.
4. With center at F draw arc AG. With center at B draw arc GH.
5.  $AB = 1$ ;  $BH = \phi = .618\dots$ ;  $AH = \phi^2 = .382\dots$

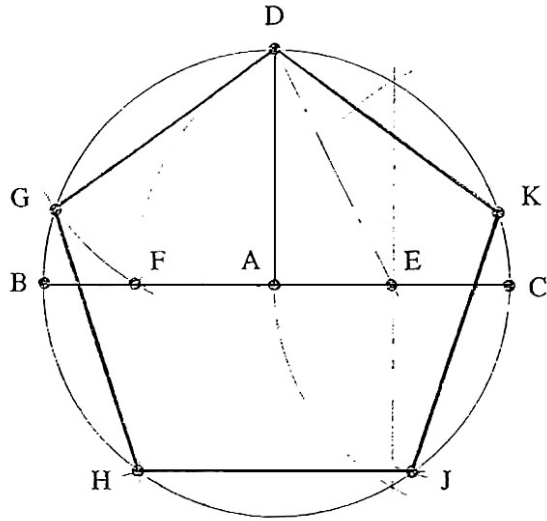
6b - Divide a Line by the Golden Section

Defined mathematically  $\emptyset = [\sqrt{5}-1]/2$ ; beginning with the square root of five,  $\sqrt{5}$ , which equals 2.236..., subtracting unity we have 1.236...; and dividing by 2 gives .618... .  $\emptyset$  is the only expression that satisfies the equation  $a+a^2=1$ , that is, root plus square equals unity. Phi is the only ratio that satisfies this condition. as  $.618... + .381... = .999... \approx 1.0$ . Phi, as a transcendental, does not actually reach 1 but makes what might be called a 'near approach'. The Phi ratio, in creating a progression in  $\emptyset$  proportion, can have two values: 1:1.618... if the progression is increasing and 1: .618... if the intervals are decreasing. For the sake of consistency  $\emptyset$  will be taken as .618... unless otherwise noted.

Just as the golden sun and the metal gold are symbolically related to unity the golden section embodies an approach to oneness through geometric construction. To construct increasing intervals, **figure 6a**, the side of a square is set equal to 1. The semi-diagonal ED of the square =  $\sqrt{5}/2$ . The semi-diagonal is then applied on the extension of AB to find point F. The extension BF is equal to  $\sqrt{5}/2$  minus 1/2.  $\sqrt{5}/2 = 1.118... - .5 = .618... = \emptyset$ . To obtain the next interval construct a square on side AF.  $AF + FG = 1.618...$

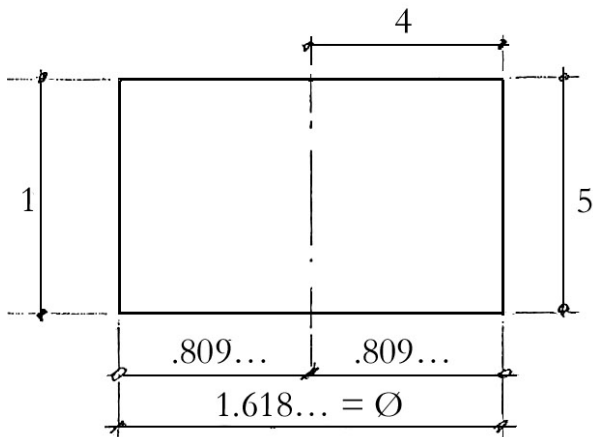
To construct decreasing intervals the side of the square is divided at  $\emptyset$ , **figure 6b**, by constructing semi-diagonal FB. With center at F and radius FA, find point G. With center at B and radius BG, draw arc GH. If  $AB = 1$ ,  $BH = \emptyset$  and  $HA = \emptyset^2$ .

The relationship of the Pentad and  $\emptyset$  is shown in **figure 6c**. Just as the diagonal of the square is related to the  $\sqrt{2}$  and the altitude of the equilateral triangle is related to the  $\sqrt{3}$ ; if the side of the pentagon is equal to 1, its diagonal is  $1+\emptyset = 1.618...$  **Figure 6d** shows the derivation of two  $\emptyset$  related ratios. Half of a 1:1.618... ratio gives a ratio of 1:.809..., a small whole number approximation of which is 5:4. Doubling a ratio of 1:.618... gives 1:1.236... or approximately 5:6.

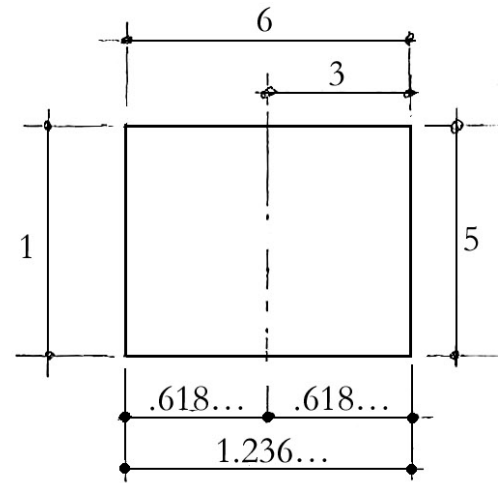


1. Given a circle with center A and diameter BAC.
2. Erect a perpendicular from A to find D.
3. Bisect AC to find E.
4. Draw arc with center at E and radius ED to find point F.
5. With center at D and radius DF draw arc FG.
6. Mark off interval DG on the circumference to find points H, J and K. DGHJK is a pentagon.

6c - Construct a pentagon within a circle.



The 5:4 ratio is created by dividing a Golden Rectangle



The 5:6 ratio is created by combining two Golden Rectangles

6d - Two ratios derived from the Golden Section

### Figure 7. Squaring the Circle

Vitruvius<sup>2</sup> prefaces his method of designing temples by saying "*Without symmetry and proportion there can be no principles in the design of any temple; that is, if there is no precise relation between its members, as in the case of those of a well shaped human body.*" A little later he adds in illustration, "*if a man be placed on his back, with his hands and feet extended, and a pair of compasses centered at his navel, the fingers and toes of his two hands and feet will touch the circumference of a circle... [and] ...if we measure the distance from the soles of the feet to the top of the head, and then apply that distance to the outstretched arms, the breath will be found to be the same as the height...*" thus forming a square.

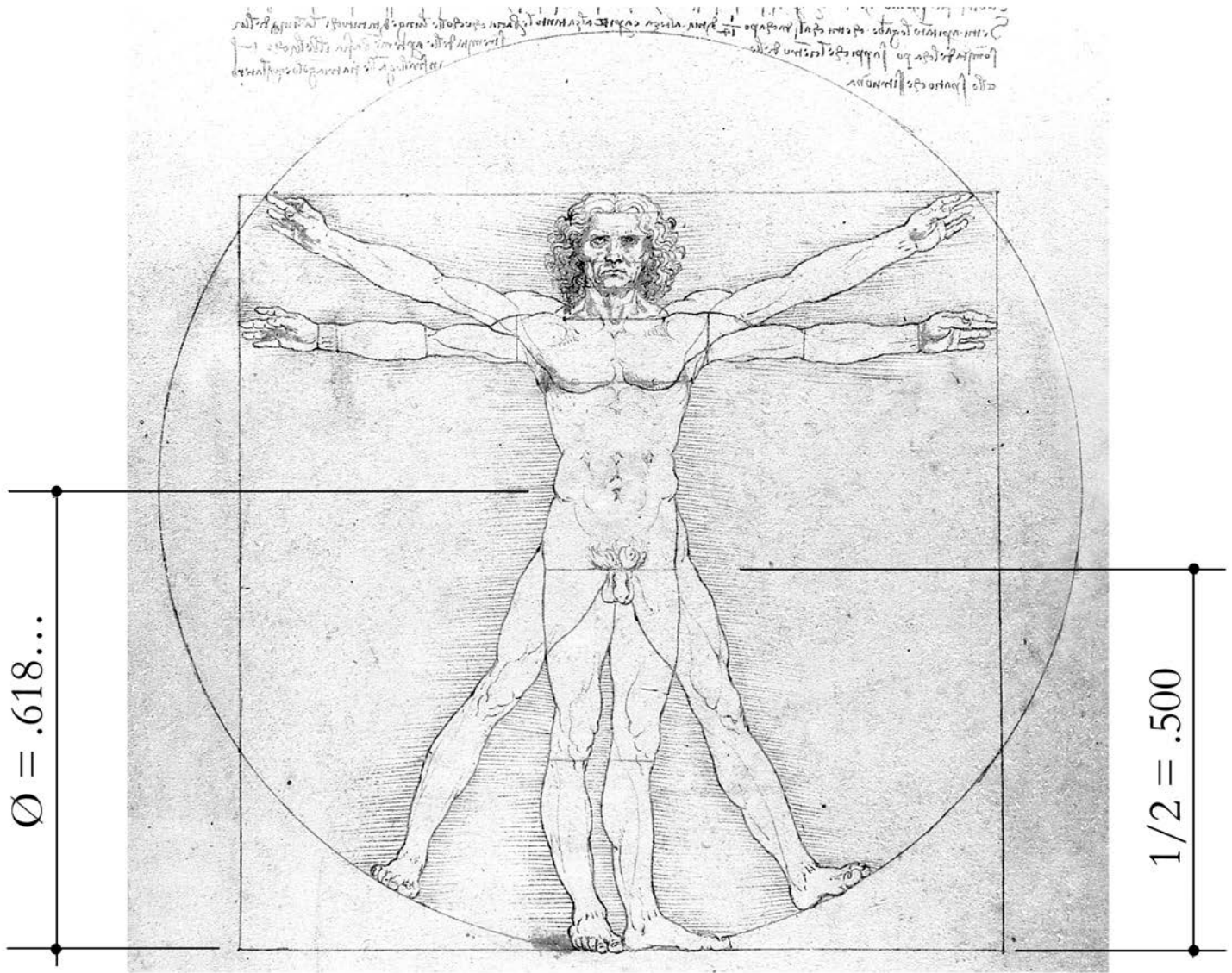
In the geometrical creation story, **figure 1**, the circle is a symbol of transcendent unity and the square is a symbol of unity in the material realm. In the Pythagorean-Platonic world view our lives as intelligent human beings result from a kind of intimate or loving embrace between spirit, the circle, and matter, the square. This concept could be symbolized by creating a relationship of equality, a proportional relationship, between a square and a circle.

To construct a circle and square that have equal perimeters set the side of the square equal to 1. Its perimeter will equal 4. The diameter of a circle whose perimeter or circumference equals 4 is  $4/\pi$  or 1.273..., making the radius .636... as compared to 1, the side of the square. The ratio of approximately .6/1 or 6/10 may be taken as a reference to  $\emptyset$ . Its geometric construction is shown in **figure 5**. As the name suggests  $\emptyset$  is not a number but rather a relationship, a ratio. In practical terms it is a multiplier used to relate other magnitudes to achieve an equality of ratio, that is to say, proportionality.

For example, 1:2::4:8 is a proportion in double ratio; that is, the ratios are equal, in each ratio the second term is double the first. There may also be a proportion in  $\emptyset$  ratio. While any number can be used to create a proportional group of ratios,  $\emptyset$  occupies a special position which may be stated mathematically as  $\emptyset + \emptyset^2 = 1$ . This may symbolize the interweaving of matter and spirit; where  $\emptyset$  represents spirit, the unmanifest, and  $\emptyset^2$  represents matter, the manifest. Together they equal 1, unity, which may also represent life and intelligence comprising the soul.

This relation to unity also implies that the individual and the cosmos are created on the same pattern, that they are constructed by analogy. Implied in Vitruvius' discussion, then, is the idea that the temple or perfect building is a living mediator between the individual and the cosmos. Proportion, in this context, can be seen as the soul of the building, uniting and vitalizing the idea of the building with its physical body. This Vitruvian comparison of the building and the body, called 'anthropos', was visualized in the Renaissance by Leonardo da Vinci in his famous drawing known as the Vitruvian Man.

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Perimeter of square = side x 4 = 1 x 4 = 4  
 Circumference of circle =  $\pi \times D = 3.14\dots \times 1.236\dots = 3.88\dots$   
 $\varnothing$  squares the circle within 3%

Figure 7 - Squaring the Circle

**Figure 8. A Tetrastyle Portico**  
using  $\emptyset$  and the Squaring of the Circle

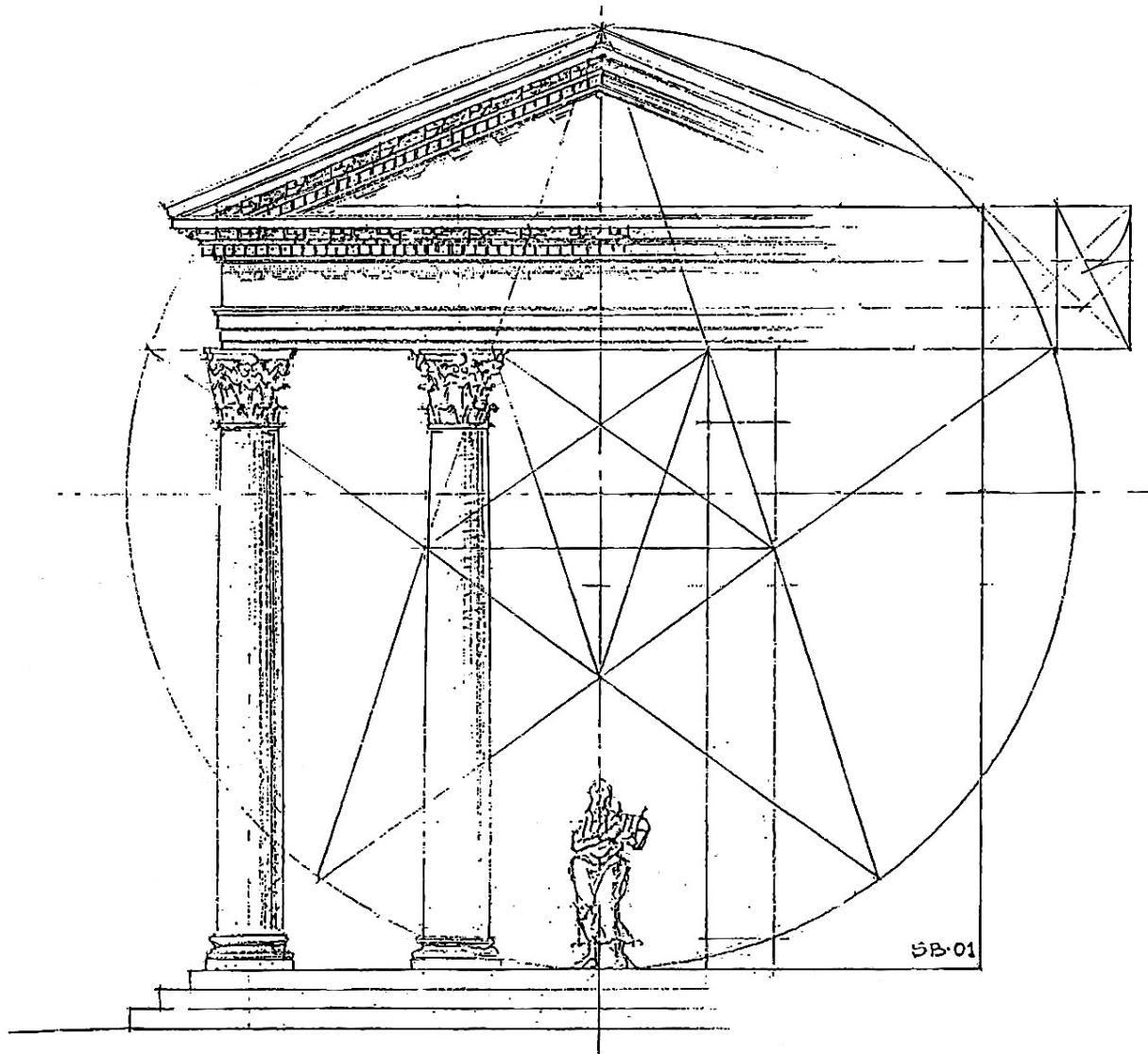
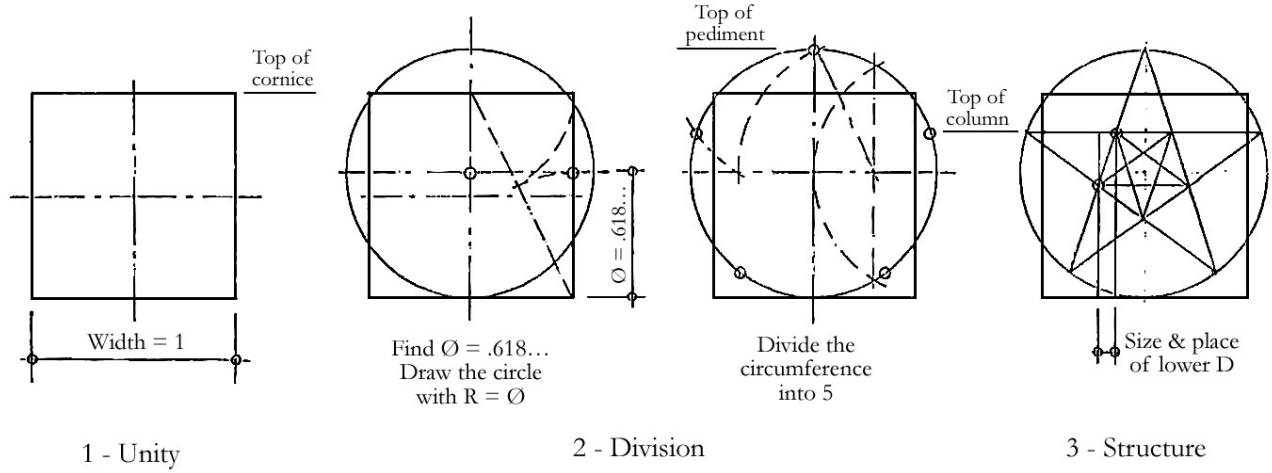
The method of division of the circle into five parts, the internal divisions of the pentagram and the squaring of the circle could provide appropriate geometrical keys for another tetrastyle design. As in **figure 3** the process can be seen to unfold in four stages:

**1. Unity** - As in the first example we repeat the steps of our ontogeny, arriving at the square.

**2. Division** - As a fundamental act of division we apply the  $\emptyset$  ratio to the height of the square and use this distance as the radius of a circle which will mark the apex of the pediment.

**3. Structure** - To obtain the size and placement of the two inner columns divide the circumference of the circle into five parts as in **figure 6c**. Connecting these points, construct the pentagram. Intersecting points on its diagonals mark the column diameter and its position. The horizontal diagonal will mark the lower edge of the entablature. The top of the original square gives the top of the entablature. The square and circle are in the relationship of the 'squaring of the circle' symbolizing a harmonic union of spirit and matter.

**4. Materialization** - Having placed and sized the major elements we now complete the elevation and move to detailing the elements. We select the Corinthian order for this design because of its inner relationship to the pentagon and phi. This is hinted at in the myth of Callimachus as told by Vitruvius in which we see the artist meditating at the grave of the 'Corinthian Maiden'. His design for a capital is inspired by leaves, flowers and volutes which have grown up around a basket of the Maiden's grave objects, which was weighted down with a square stone tile. The 'maiden' may be taken as a reference to Venus-Aphrodite. Though outwardly the spirit of the maiden has taken the form of vegetation, the inner structure of the capital's composition may be based on the double pentagon, suggesting that  $\emptyset$  has been the medium of this transformation.



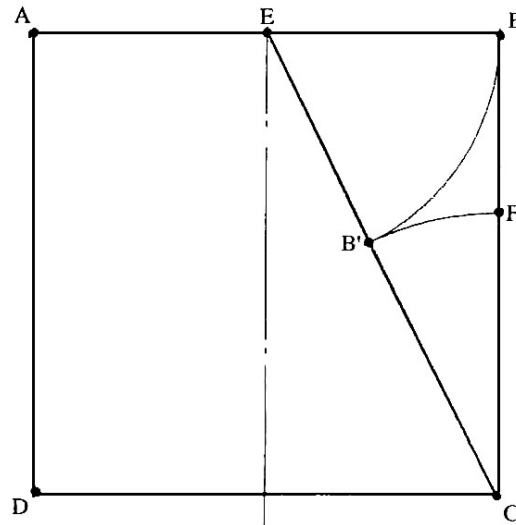
4 - Manifestation

Figure 8 - A Tetrastyle Portico using  $\phi$  and the squaring of the circle

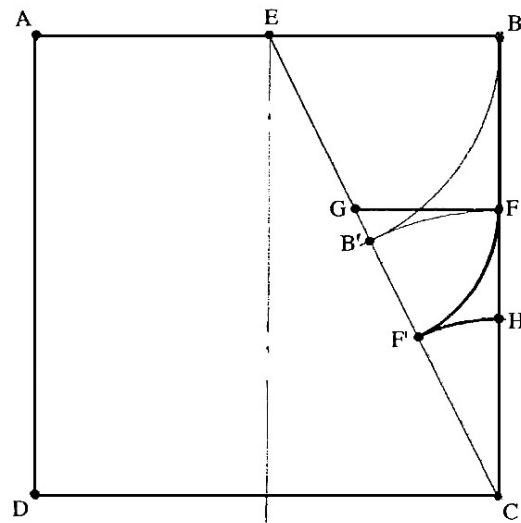
**Figure 9. Creating a  $\emptyset$  Scale**

Beginning with an interval defined as unity we may continue the procedure of **figure 6b** to construct a series of  $\emptyset$  related intervals, or a  $\emptyset$  scale.

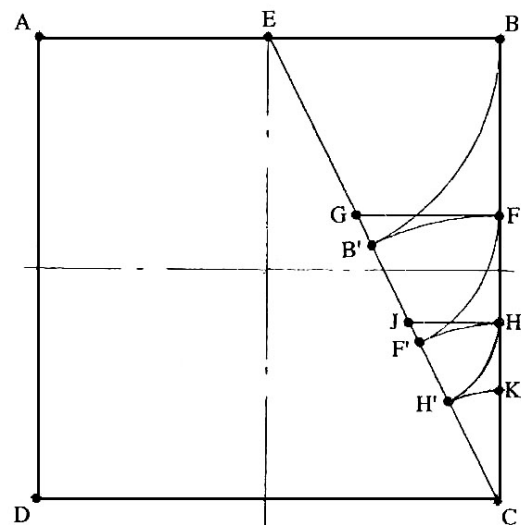
1. Given square ABCD with E as the midpoint of AB. Draw semi-diagonal EC. With center at E and radius EB draw arc BB'. With center at C and radius CB' draw arc B'F. CF is  $\emptyset$ , .618... of BC.



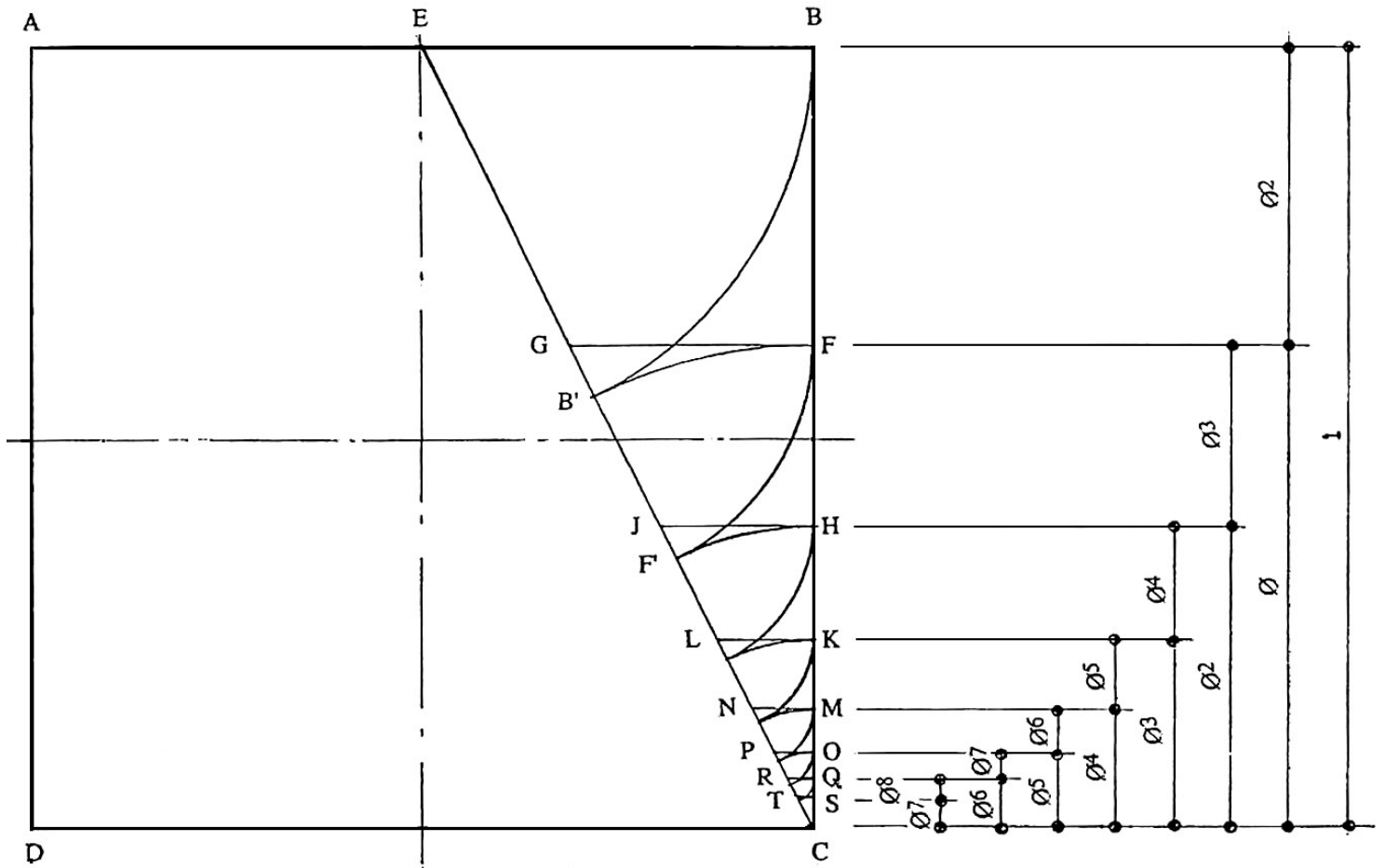
2. Draw line FG parallel to AB. Repeating the procedure in step 1, with center at G and radius GF draw arc FF'. With center at C and radius CF' draw arc F'H. HC =  $\emptyset^2$  and FH =  $\emptyset^3$ .



3. To obtain the next interval we repeat the procedure. Draw line HJ parallel to AB. With center at J and radius HJ draw arc HH'. With center at C and radius CH' draw arc H'K. CK =  $\emptyset^3$ , KH =  $\emptyset^4$ .







4. The scale may be carried on indefinitely, to the limit of practicality. Here we have gone to  $\text{Ø}^8$ .

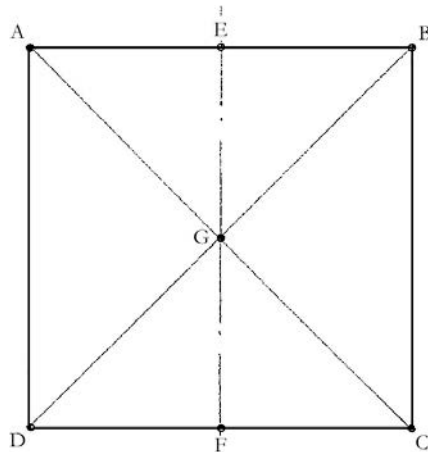
**Decimal Equivalents of the Ø Scale**

$\text{Ø} = .618$	$\text{Ø}^5 = .090$
$\text{Ø}^2 = .382$	$\text{Ø}^6 = .056$
$\text{Ø}^3 = .236$	$\text{Ø}^7 = .034$
$\text{Ø}^4 = .146$	$\text{Ø}^8 = .021$

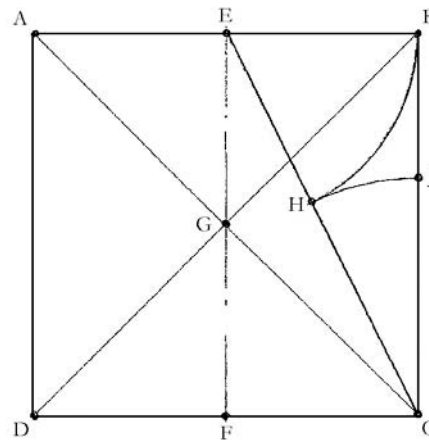
**Figure 10. A Tetrastyle Portico**  
using  $\emptyset$  and the  $\sqrt{2}$

Set up a square as in the previous examples but here subdivide it with a  $\emptyset$  scale, as in **figure 9**. Setting the side of the square as 1,  $\emptyset = .618\dots$  and  $\emptyset^2 = .382\dots$ .  $\emptyset^2$  divided in half,  $= .191\dots$ , is the depth of the entablature.  $\emptyset$  gives the position of the inner columns. The diameter of the columns is given by  $\emptyset^5 = .090\dots$ , but may also be given by the difference between  $\emptyset$  and the  $\sqrt{2}$ ,  $.707\dots - .618\dots = .089\dots$ . As the height of the columns in this design is  $.809\dots$ ,  $\emptyset^5$  gives a ratio of 1:9 and  $.089\dots$  gives 1:9.1. Vitruvius gives a ratio of 1:9.5 and Vignola gives 1:9 for the Ionic order. D of  $.090\dots$  is 1/11th of the width of the portico; D of  $.089 = 1/11.24$ ; Vitruvius divides the width into 11.5 parts; the design in **figure 3** used division into 12 parts. The height of the pediment is set by an interval of  $\emptyset^2/2$ , giving an overall height of 1.191... or approximately a width to height ratio of 5:6, recalling the squaring of the circle, **figure 7**.

1. The portico is constructed within square ABCD, it's centerline is EF with center at G.
2. Draw the semi-diagonal EC. With center at E and radius EB draw arc BH. With center at



10.1

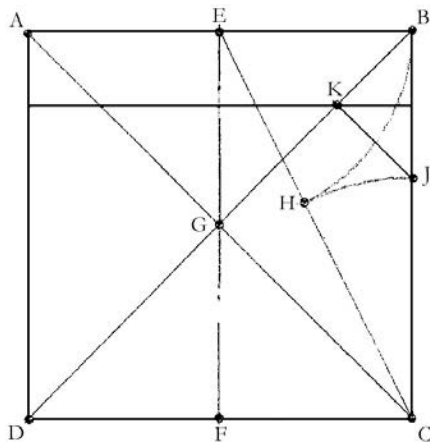


10.2

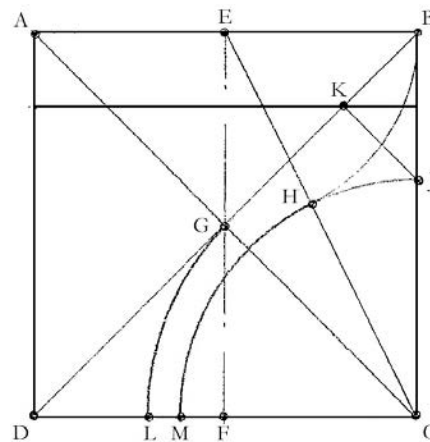
C and radius CH draw arc HJ. If  $BC = 1$  then  $CJ = \emptyset = .618\dots$  and  $BJ = \emptyset^2 = .382\dots$

3. Divide BJ in half to find point K.  $BK = \emptyset^2/2 = .191\dots$   $CK = .618\dots + .191\dots = .809\dots$  marking the top of the columns.

4. With center at C and radius CG draw arc GL. With center at C and radius CH draw arc HM. LM gives the location and size of the column.



10.3



10.4

If we associate  $\sqrt{2}/2$  with the coming forth into light of Apollo and  $\emptyset$  with the darker Hermetic mysteries then we see the column diameter as Hermes and Apollo working together. In the greek myths Hermes steals the solar cattle of Apollo and hides them in a cave, where the sun does not shine. He sacrifices one of the cows and uses its parts to create the musical instrument the lyre. Apollo eventually finds the cattle and brings the theft to Zeus for settlement. Zeus returns the cattle and assigns the lyre to Apollo. The lesson for us is that the unlimited creativity of Hermes must be civilized, that is, limited by Apollo who also governs the Muses

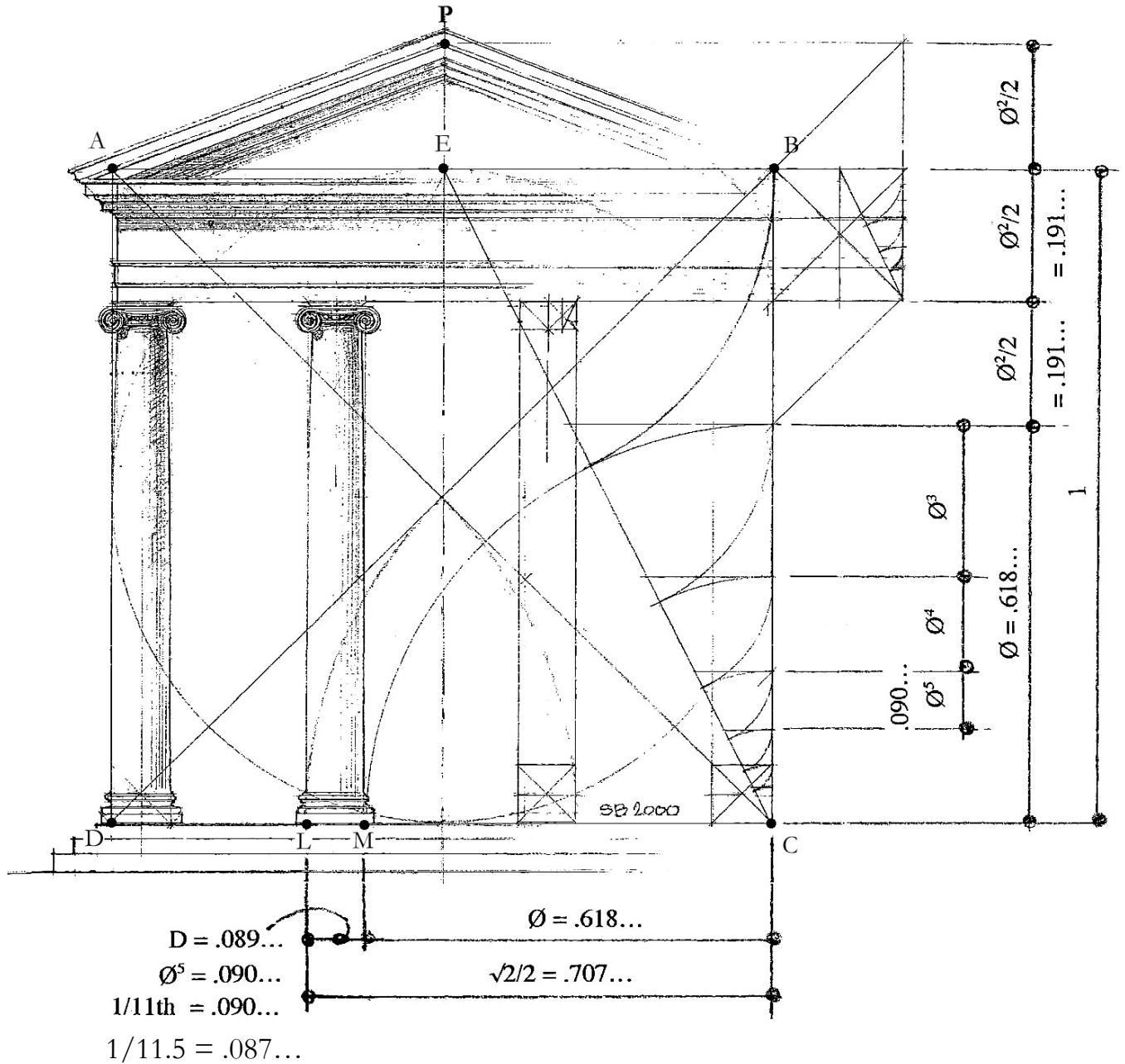
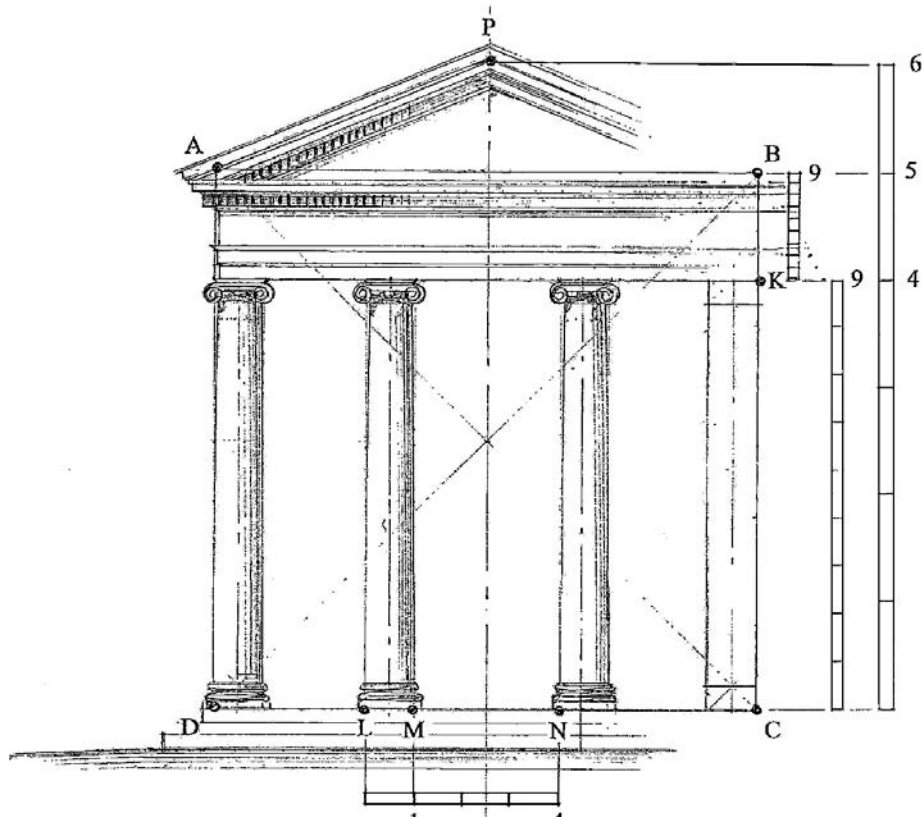


Figure 10.5 - A Tetrastyle portico

**Figure 11. A Tetrastyle Portico**  
using harmonic ratios



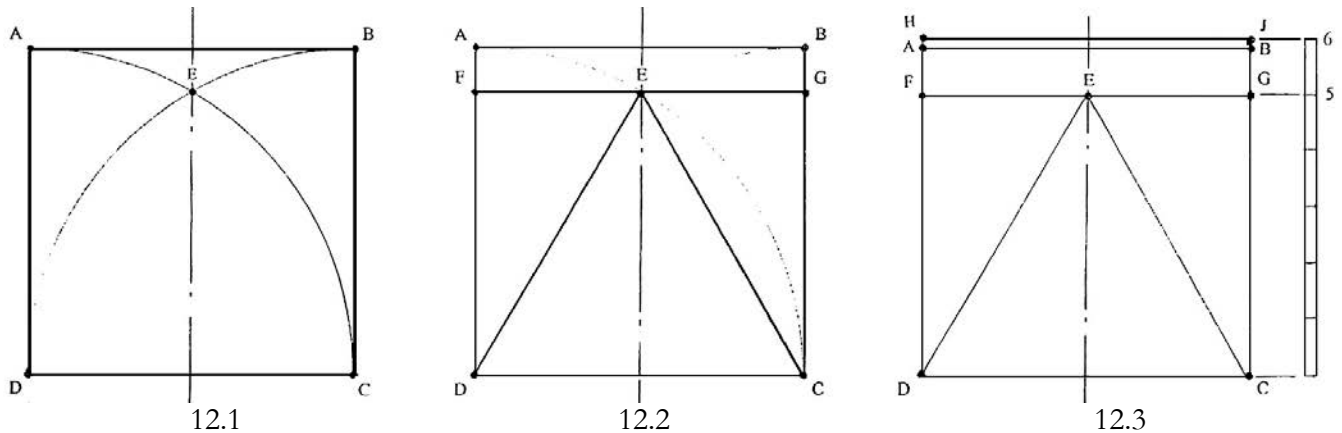
Constructing a four column portico using the of harmonic ratio method begins with square ABCD. Divide its height BC into 5 parts. The topmost interval BK will be the entablature. This makes the shape under the entablature a 5:4 rectangle. The apex of the pediment will be one part higher, point P, fitting the elevation as a whole into a 6:5 rectangle. Both these ratios are phi related; 5:4 is half of a phi rectangle and 6:5 results from addition of two phi rectangles, **figure 6d**.

To size the columns divide the 4 lower parts, interval CK, into 9 units. The column diameter, LM, will be 1 unit; the central intercolumniation, MN, will be 3 units. While the column height complies with Vignola's canon, the 4:9 relation may not seem harmonious but it is also phi related. The 4:9 ratio may be expressed as 1:2.25, a very close approximation to  $1:\sqrt{5}$ , 1:2.236... . If the 9 unit scale were extended up to the top of the square it would measure 11.25 units, very close to the Vitruvian arithmetic division of the tetrastyle portico into 11.5 units<sup>3</sup>.

If the central intercolumniation is 3 units, and allowing 4 units for the diameters of the columns, that leaves 4.25 units for the two side intercolumniations, or 2.125 units each. Thus interval MC would equal  $3 + 1 + 2.125 + 1 = 7.125$ ;  $7.125/11.25 = .633...$ , very close to  $\phi = .618...$ , which marked the location of point M in **figure 10**. The entablature in the canon of Vignola is divided into the same number of parts as the column, in this case 9. The architrave is 2.5 parts, the frieze is 3, and the cornice is 3.5.  $\phi$  divides 9 units at 5.5 and 3.5. While we could go on citing such close approximations, the point is that the geometric, arithmetic and harmonic ratio methods may be used to produce canonical structures and elements.

**Figure 12. A Tetrastyle Portico**  
using the  $\sqrt{3}$

Previous tetrastyle examples have utilized the musical ratios of the tetrachord, the  $\sqrt{2}$ , and  $\emptyset$  related ratios to construct porticos. In this example the portico is guided in part by the equilateral triangle. It creates a higher column and lighter entablature, giving an elegant elongated character.

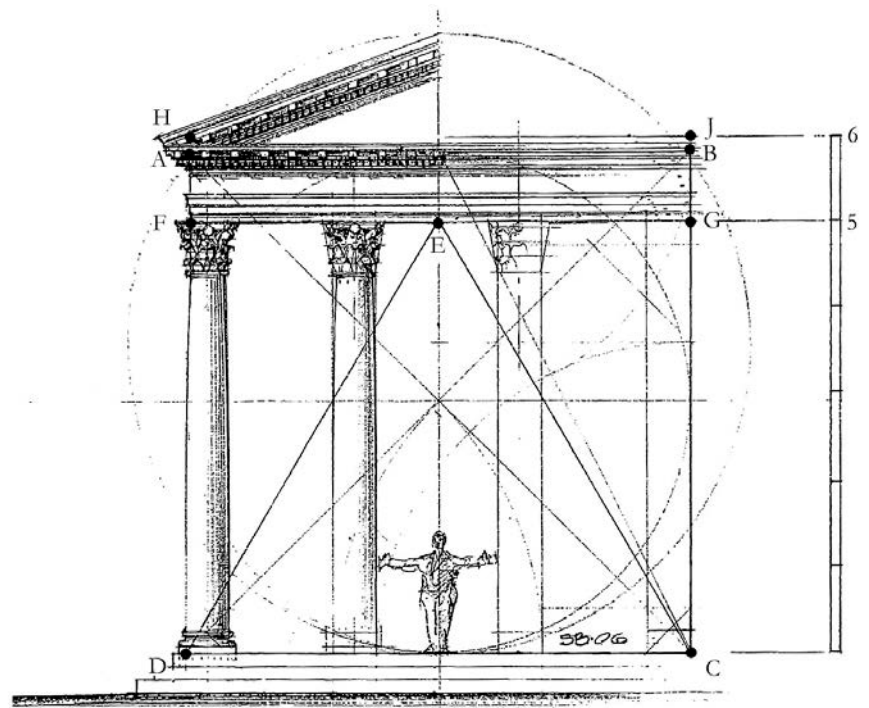


**12.1.** Begin with square ABCD. DC will be the width of the portico. With center at C and radius CD, draw arc DB; with center at D and the same radius draw arc CA. Their point of intersection is E, the apex of an equilateral triangle.

**12.2.** Draw line FG through E, parallel to AB. Line FEG sets the height of the columns.

**12.3.** Divide the column height into 5 parts. Add an additional part to create interval GJ, giving the depth of the entablature. The top of the entablature is HJ which is slightly higher than the top of the square AB.

**12.4.** The column diameter and placement may be done according to the method of **figure 10**. If column D is  $1/11$ th of the width or .089... and the column height =  $\sqrt{3}/2$ , the altitude of an equilateral triangle or .866..., then  $H=9.75 D$ ; or the height may be simply divided into 10 for a D of .086... These ratios are within the canonical range for the Corinthian order as given by Vitruvius [ $D = 9.5 H$ ] and Vignola [ $D = 10 H$ ]. Vignola provides for an entablature to column ratio of 1:5 as well as 1:4. The pediment may be added according to any of several methods.



12.4

**Figure 13. A Hexastyle Portico**  
using  $\emptyset$  and the  $\emptyset$  scale

The hexastyle portico may be constructed using  $\emptyset$  as a key, following the four steps in our ontological myth, **figure 1**, Unity, Division, Structure and Manifestation.

**Figure 13.1:** Draw a circle whose diameter is the width of the portico and set it equal to 1. Enclose the circle, representing spirit or inner Unity, in a square, ABCD, representing matter, the manifest or outward Unity. The half point of the square, line HF, will be the height of the columns. HFCD is a 2:1, or octaval rectangle.

**Figure 13.2:** Apply a key pattern of division, internally dividing the square by  $\emptyset$  using the method of **figure 6b**. Draw semi-diagonal EC. Apply radius EB on EC to find point B'; with center at C and radius CB' draw arc to find point J. Apply arc CJ to the base CD to find point K.  $CJ = \emptyset = .618\dots$  As  $CF = .500$ ,  $JF = .118\dots$  or  $\emptyset^3/2$ .

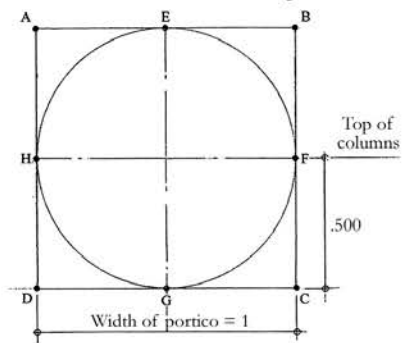
**Figure 13.3a:** Place the elements of structure. JF, .118..., will be the depth of the entablature. Continue to develop the phi scale by the method of **figure 9** to find point L.  $CL = \emptyset^2 = .382\dots$  Applying this interval along CD gives point M. Points K and M are the centers of the inner pair of columns. The interval between them  $KM = JL = \emptyset^3 = .236\dots$

**Figure 13.3b:** Continue the phi scale to find points N, P, Q and R. The interval  $CR = \emptyset^6 = .056\dots$  This will be the lower diameter of the columns. Interval PQ is also equal to  $\emptyset^6$ , as  $\emptyset^5[CQ] + \emptyset^6[QP] = \emptyset^4[CP]$ , thus construction of the scale need only be pursued to point Q.

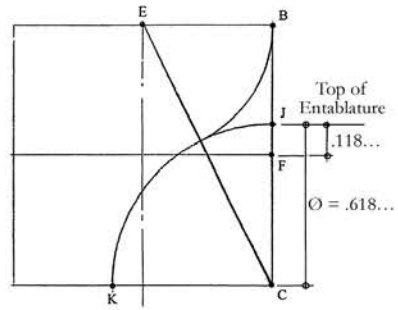
The last element of structure is the pediment. Its apex is found by bisecting interval BJ to find point S.  $BJ = \emptyset^2 = .382\dots$ ; divided by 2 = .191... The overall height of the portico will be  $SG = .618\dots + .191\dots = .809\dots$

**Figure 13.4:** To complete the portico, locate the outer columns at the perimeter of the square and the middle columns equally between the inner and outer. The capital and base heights are taken to be 1/2 of lower D. The entablature is subdivided by the same pattern of  $\emptyset$  division as the whole portico.

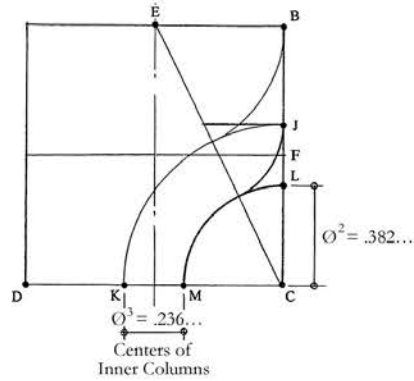
This construction falls within the canonical range established by Vitruvius and Vignola. Both of these systems will be taken up in later sections but for the moment note that Vitruvius divides the width of the six column portico into 18 units;  $1/18\text{th} = .0555\dots$  and that our lower D =  $\emptyset^6 = .0557\dots$  This interval compared to the height of the columns, .500, gives a D/H ratio of  $1/8.98\dots$  where Vignola gives 1/9 and Vitruvius gives 1/9.5. The Vitruvian entablature = 1.93 D. Our entablature is  $.118 \div .056\dots = 2.11\dots$  D. Vignola gives 2.25 D when the entablature is 1/4 of the column height but he also says the ratio may be 1/5 which gives a ratio of 1.8 D. The central intercolumniation given by Vitruvius is 3D. Ours is  $.236\dots - .056\dots = .180$ , or 3.21D. Thus, the major elements of our construction falls within the canonical range established by Vitruvius and Vignola.



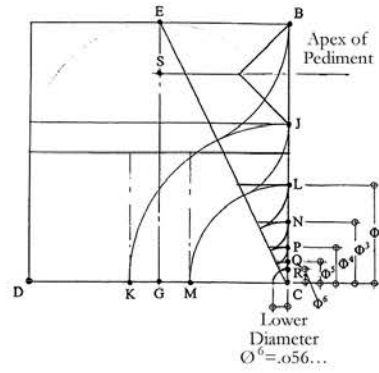
13.1 - Unity and its Representatives



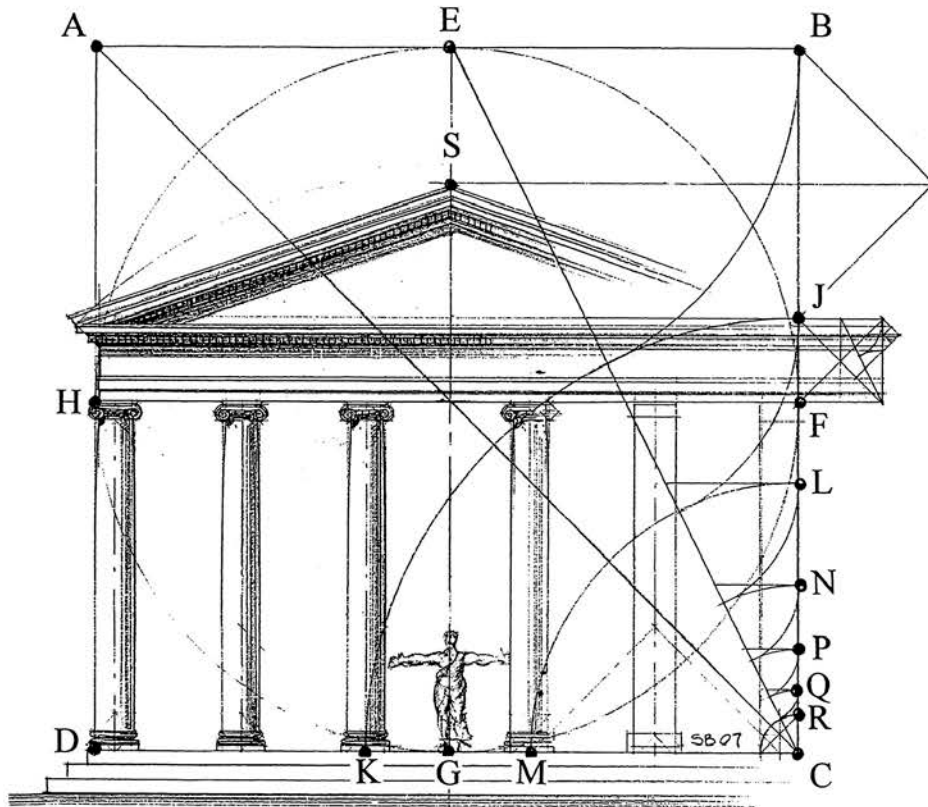
13.2 - Division, the Major Cuts  
The Double Square and  $\phi$



13.3a - Structure, Placement of the Inner Columns

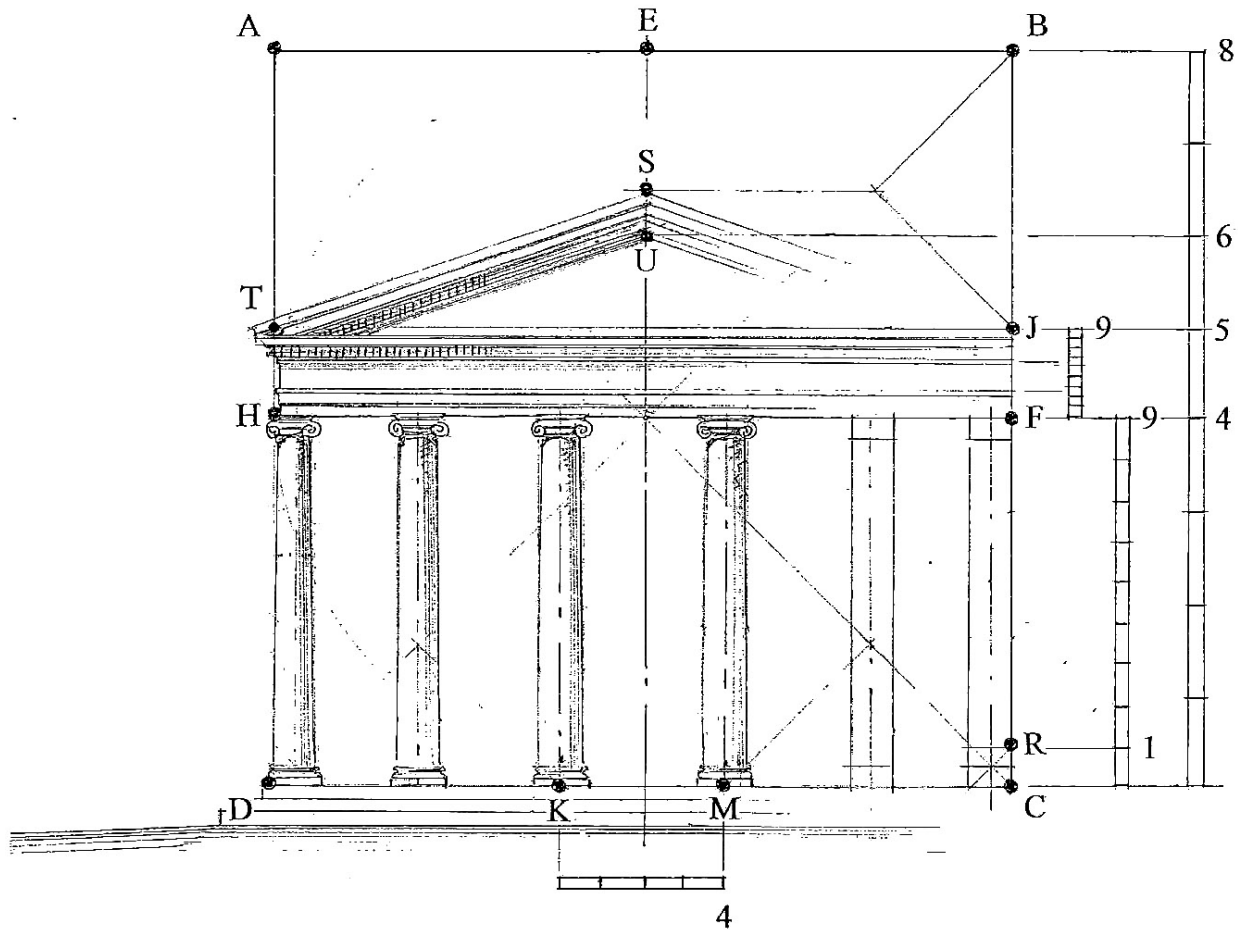


13.3b - Structure Column Diameter



13.4 - Manifestation  
Ornamenting the Elements

**Figure 14 - A Hexastyle Portico**  
using harmonic ratios



The six column portico can be constructed with harmonic ratios as was done with the four columned portico, **figure 11**. Begin with the square ABCD and the half square HFC. Point F will mark the height of the columns. Divide CF into 4 parts; adding a fifth part will give the depth of the entablature. Adding a sixth part will give the apex of the tympanum, U; or dividing the upper three units, interval BJ, in half will give the apex of the pediment, point S. As in **figure 11**, the column height, CF, is divided into nine parts; each will be 1 diameter; the central intercolumniation will be three diameters. Since CF is half the side of square ABCD the width of the portico, CD may be divided into 18 column diameters, in exact agreement with Vitruvius' division.

The entablature is divided into 9, or if one wishes, into 18 parts. In terms of ratios, the colonnade is 8:4 or 2:1, the double square and the ratio of the octave; the rectangle CDTJ is 5:8 or  $\frac{5}{8}$ . The top of the tympanum, point U is at the  $\frac{6}{8}$  or  $\frac{3}{4}$  point, the musical fourth and one of Palladio's recommended room ratios. Each side intercolumniation is 2.25 column diameters. Interval CK is  $\frac{11}{18}$  or  $\frac{11}{18}$ , as it was in **figure 13**.



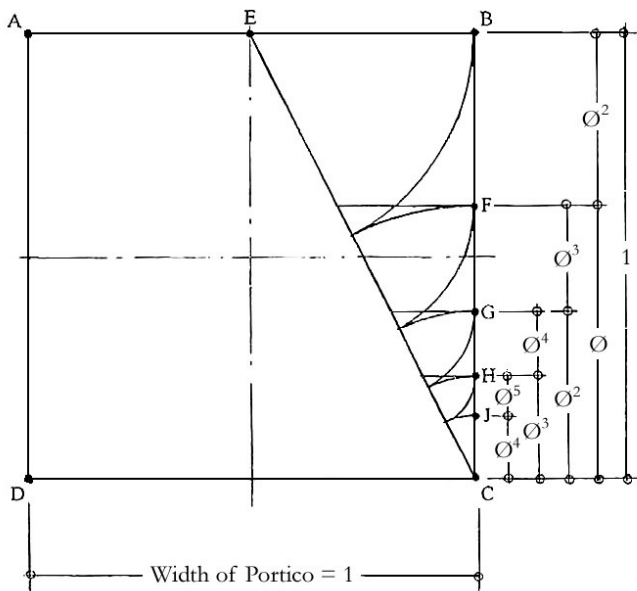
### Figure 15. An Octastyle Portico using a $\emptyset$ Scale

**Figure 15.1-2.** To construct the octastyle portico, follow a similar procedure as in figures 10 and 13. Set up square ABCD as the representative of unity. AB, set equal to 1, will equal the width of the portico. The key pattern of division is once again the  $\emptyset$  scale, but we will have to be more flexible in its application.  $\emptyset$  intervals will be divided in half and taken out of sequence, translating them to where they are required.

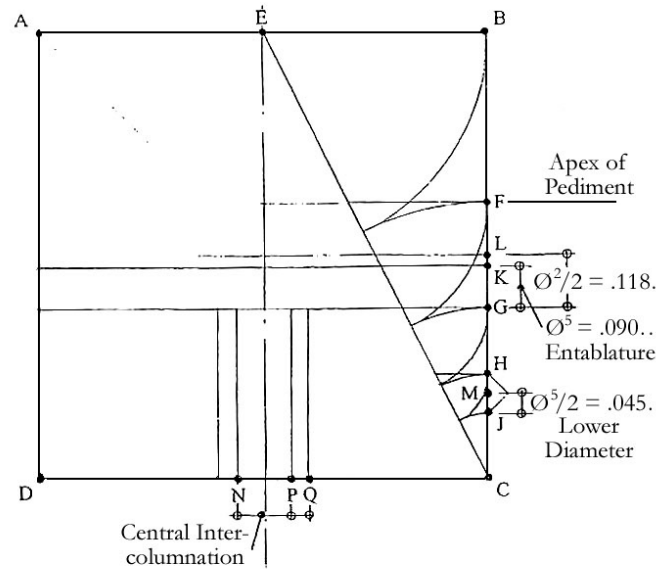
**Figure 15.3-4.** In structuring the elements, the  $\emptyset$  interval, FC = .618..., will be the overall height of the portico to the top of the pediment. The  $\emptyset^2$  interval, GC = .382..., will be the height of the columns. The depth of the entablature will be  $\emptyset^5$ , interval HJ = .090... . Apply this interval by placing it atop point G. The central intercolumniation will be the interval NP = GL =  $\emptyset^3/2$  = .118....

The diameter of the columns will be PQ = HM,  $\emptyset^5 \div 2 = .045...$  . With a column height of  $\emptyset^2 = .382...$  the diameter to height ratio [D/H] = 8.5. Vignola gives 9.0 which would give a D of .042...; Vitruvius gives 9.5 or a D of .040. The pure geometry differs from the Vitruvian arithmetic canon by an interval of .005.... This is 1.3% of the column height, .382...; or 0.5% of the overall width of the portico.

The center intercolumniation, .118..., is 2.6 D. Vitruvius gives 3 D or .120. The entablature is .090 in depth. In the Vignolian canon it would be 1/4 of the column height, .382..., or .095...; in the Vitruvian canon it would be 1.95 D or .078 differing by .017, or 4.5% of the column height. This comes within 5% of Vitruvius construction and considerably closer to Vignola's.



15.1-2 - Unity & Division

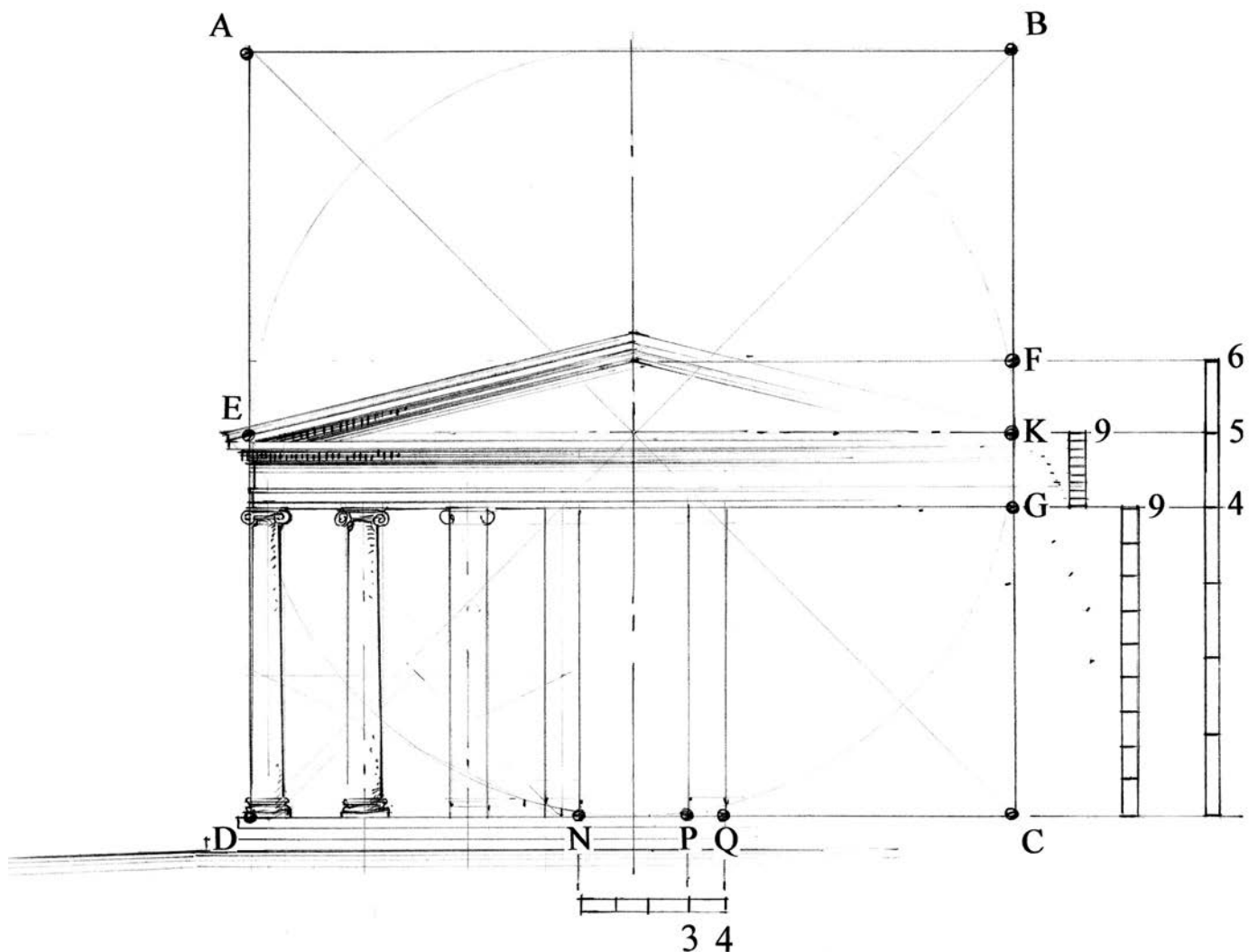


15.3 - Placement of Elements



**Figure 16. An Octastyle Portico**  
using harmonic ratios

To construct an octastyle portico using small whole number ratios begin with square ABCD. Divide it in half to create a 2:1 rectangle, EKCD. Divide the height CK into 5 parts; the columns will be 4 parts high, CG, and the topmost part will give the location and depth of the entablature, GK; a 6th part will place the apex of the tympanum, point F. Divide the 4 parts of the column height into 9 units, each of which will be 1 column diameter. The central inter-columniation will be 3 diameters, NP. Column D will be 1 part, PQ, and equal  $.044\dots$  of the overall width of the portico; dividing it into  $22\frac{3}{4}D$ . Vitruvius gives  $24\frac{1}{2}D$ . If we had divided the column height interval into  $9\frac{1}{2}$  parts D would be  $.042\dots$ , coming even closer to the Vitruvian  $.040$ .



### Closing

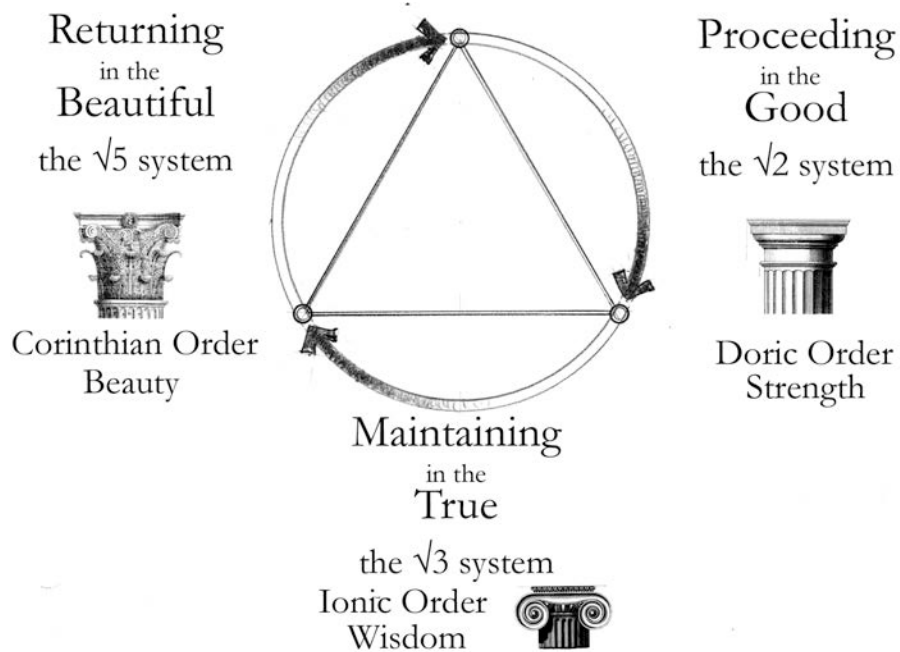
With this construction we've reached a stopping point. In these sessions we've utilized number and geometry in qualitative and symbolic terms, not as quantities or as mathematics. We illustrated a theory of design unfolding in four stages - unity, division, structure and matter. We looked at rational ratios like the octave; and the transrationals like phi. These ideas were applied by three methods - geometrical, arithmetic and harmonic or small whole number ratios. I would urge that you do try the various constructions at home.

But most importantly the goal of the classical is the creation or allowance of the possibility of the experience of beauty

Beauty is the memory of Unity - 'the joyous state of the soul as it recalls its relation to divine Unity'. It is a kind of anamnesis. Beauty is not in things or the measure of things - it's in you - it's an internal experience for which one needs preparation. Some people are more prepared than others leading to the modern notion that beauty is subjective but we've offered an objective criteria - the memory of unity. This can be thought of as having four stages, as per Diotema's lecture:

- \* transcendent beauty
- \* \* moral beauty
- \* \* \* intellectual beauty
- \* \* \* \* physical beauty

### Cycle of the Good, the True and the Beautiful



There is also a three part cycle of existence. The cosmos is said by Plato's Timaeus to proceed from the 'Good' because the Divinity is essentially Good and wanted everything that possibly could exist to experience that Goodness, thus creating space and time. The cosmos maintains in the 'True', that is the Law or the Way in Daoism, called Dharma in India or maat in Egypt. The cosmos returns in the Beautiful because as we have seen Beauty is the memory of Unity. In the illustration the three principle Orders of architecture, Doric, Ionic and Corinthian, are related to these aspects. Doric in the coming forth or budding of the Echinus, Ionic in the unrolling of the scrolls of the law, and the Corinthian capital in the rebirth or return of the Corinthian maiden.

But as artists we should not be frustrated in the pursuit of beauty - we might want to let others be the judge if it has been achieved. - We're in the situation of Hephaestus the god of artisans who is also the husband of Aphrodite, goddess of physical love and beauty. it's an arranged marriage set up by Hera who doesn't especially like either one. Hephaestus is crippled and can only walk with assistance. He is like the Egyptian god Ptah who governed coming into form; and Hephaestus is the least pretty of the gods. Aphrodite's job is to make love and she does everyone except of course Hephaestus. Finally, Hephaestus goes around the bend and tricks his wife into a date with her regular boyfriend Aries. While they are in bed together he throws a fine bronze wire net over them and calls the other gods to witness his plight. The other gods think this is very funny but ultimately its Hephaestus who is demeaned. -

Something important is revealed here - that the artisan is married to beauty - and bound to seek it - but beauty cannot be owned or controlled we can never possess it - but must always seek it.

Now if beauty is the goal, proportion is the tool that is used to create the 'sense of Unity' that opens the way to the experience of beauty. Possession of a tool conveys a power - any such power must be used for the good.

Remember that proportion is not the subject of art - it is the method it is the background or inner structure for 'mimesis'- imitation of nature and the beginning of historia or character. but mimesis must be in the service of anamnesis. if it is to allow for the beautiful

Lastly, the experience of beauty produces Eros - desire - desire to join with the object of beauty. this applies in physical life as well as art.

The sense of unity creates a bond of desire between artist and observer. underlying this connection is revealed the presence of Love - the bonding force that holds the element of the cosmos in place. If I drop the pen it falls to the earth. the modern explanation is 'gravity' but the ancient explanation is that the pen falls to earth because it loves the earth and the earth accepts it because it loves the pen.

But as I don't know anything you don't already know, my task is only to help you remember. To explore these more profound mysteries of Love I'm not qualified to lead -but will have to come down from the podium and join with all of you on the portico of that great temple of Love and Beauty.

Thanks for your interest in these subjects and for accompanying me thus far.

Good luck on your journey.

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